Libertas Mathematica (new series)
Volume 38(2018), No. 1, 1-27

About the efficiency behavior of the Portuguese manufacturing firms during the financial crisis

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Abstract: This work studies some effects of the World Financial Crisis on firms in terms of efficiency scores, by measuring how 23K units used inputs and produced outputs, obtained from a set of Portuguese manufacturing firms on three time periods: pre-crisis (2006-2008), pre-troika (2009-2011) and troika (2012-2013). We adopt a non-parametric approach, which combines Multidirectional Efficiency Analysis (MEA) with other techniques as cluster analysis, principal component analysis and dimensionality testing, to examine three empirical hypotheses: H(1) the performance of the portuguese firms in the manufacturing sector has been adversely affected by the financial crisis felt in Portugal in the troika years; H(2) due to the financial crisis, the manufacturing sector acquired long-term debt deliberately; and H(3) the financial crisis has affected substantially the food subsector. The results indicate that H(1) is confirmed, but not totally, H(2) is confirmed and H(3) is rejected. We also found surprisingly good affine fittings between inputs and capital, and outputs and EBIT with really good p-values. Hence, we also propose a reduction of the dimensionality of the MEA model, when it is possible to apply model fitting. If the reduction results in one input and one output, we give procedures to visualize and compare between efficiencies scores.

Keywords: Data envelopment analysis, multidirectional efficiency analysis, clustering analysis, manufacturing sectors, world financial crisis.

MSC2010: 62-07, 91C20,90C05

1 Introduction

One of the most severe economic crises on record is still affecting the advanced world. In Europe, the financial crisis originated by the 2008 U.S. subprime mortgage transformed into sovereign debt crises, around 2010, affecting several countries. One of those countries was Portugal, which applied for a bail-out program in 2011. This program, designed and coordinated by the IMF, the European Commission and the

European Central Bank (the so-called Troika) was implemented between May 2011 and May 2014, by including several austerity policies such as tax hikes and salary cuts. As a result of both the financial crisis and the austerity policies, Portugal suffered strong economic deceleration and rising unemployment. Only recently, the Portuguese economy has started to show its first signs of improvement.

The main aim of this paper is to examine the effects of the financial crisis on the efficiency of the Portuguese manufacturing sector. In particular, we study its effects at three distinct stages: efficiency levels; efficiency patterns; and efficiency determinants. We use a data set of Portuguese manufacturing firms observed between 2006 and 2013. We are interested in analyzing the ability of firms to overcome the difficulties generated by the crisis in the short term, for this reason, our study period involves a before, during and immediately after the crisis. Thus, for some analyses, the data are divided into three time periods: pre-crisis (2006-2008), pre-troika (2009-2011) and troika (2012-2013). Because the consequences of the crisis over the efficiency of the manufacturing sector may not have been uniform, seven distinct manufacturing sub-sectors are considered throughout the paper. For similar reasons, firms are also divided into groups according to their size/scale. Relative to their larger counterparts, smaller firms tend to act differently and are affected differently in numerous aspects of their economic behaviour. For example, [22] argues that small firms differ from large firms in taxability, ownership, flexibility, industry, economies of scale, financial market access, and level of information asymmetry and [23] found that, in Belgium, the reduction in the credit supply originated by the global financial crisis had a greater impact on small firms' investment due to their lower financing capacity. In contrast to traditional approaches, we do not consider the typical sample partition between micro, small, medium and large companies (or any subset of it). Instead, we use a clustering algorithm to capture the natural structure of the data and obtain the best size-based sample partition.

The most common technique for measuring the efficiency of decision making units (DMUs) is the Data Envelopment Analysis (DEA) introduced by [7]. The DEA approach has been widely investigated and applied to many fields and industrial problems (see, e.g., [18, 8, 20]). In DEA, we may apply radial contractions of the inputs and undesirable outputs and/or apply radial expansions of the desirable outputs. However, to further assess whether the financial crisis led to changes in efficiency patterns it is useful to understand also which variables where used inefficiently. Therefore, in this work we use other nonparametric deterministic method for measuring efficiency, namely a model based on the Multidirectional Efficiency Analysis (MEA), proposed by [4]. In contrast to DEA, in the MEA approach the input reduction and output expansion benchmarks are selected proportional to the potential improvements in efficiency identified by considering the improvement potential separately in each input variable and output variable. Thus, in addition to efficiency levels, MEA allows us to investigate changes in efficiency patterns across DMUs. Examples of applications of the MEA model to examine efficiency patterns can be found in [1, 25].

When analyzing the efficiency of manufacturing firms, there are many financial and economic variables that can be used as inputs and outputs. Such implies a substantial computational effort. Here, we propose a reduction of the dimensionality of the MEA, provided that these are correlated with other input or output variables, having a relatively stable behaviour over time. We show how to make this reduction of the MEA and how to choose the most meaningful variables to perform a coherent analysis of firm's efficiency, using Principal Component Analysis (PCA) proposed in [19], together with a dimensionality test described in [21], in order to avoid loss of information or the introduction of random noise. Furthermore, we present two procedures from which is possible to visualize and make simple comparisons between efficiencies along time. In one case, we show how to make comparisons between firms with respect to specific variables using model fitting. In the other case, comparisons between groups with different levels of efficiency are done by using a technique developed by [14], which is based on the calculation of a normal distribution intersection coefficient (NC-value) that measures the overlapping of Gaussian distribution functions.

The implementation of DEA/MEA can be leave some issues and pitfalls that need to be avoided (see [10]). In this work, problems with economies of scale, percentages and other normalized data, are overcome simultaneity, considering the Variable Returns to Scale (VRS) model for the efficiency measurement of decision making units, see [6] and using the results in[12].

The remainder of this paper is laid out as follows. In the next section, discusses the MEA approach and we introduce alternative ways to visualize and compare efficiencies, which complement the analysis done by the MEA model. Next, we provide a brief explanation on how these techniques, which are common in other contexts, apply to our work. It also analyzes the relevance of firm size. In Section 3 a brief overview on the effects of the financial crisis on the Portuguese firms is given, and three hypotheses are formulated. Next, we presents the main results, which allow to analyze the three hypotheses. In Section 4, some concluding remarks are formulated.

For processing the data and obtain the results presented, we develop a general sDL package based on the R language that is available online
http://sdl-vm2.mathdir.org/docs/packages/rDATA/1.3/package.html. Note that, due to size constraints, not all 224 tables and graphs obtained regarding this work are embedded in the document. However, all of them are available online and

the reader may click on the references indicated throughout the document to access them. The reader interested in further data, tables and graphs can check the web address http://tinyurl.com/l3fwrnm.In particular, we globally use the notation @[xxx] to denote the url http://tinyurl.com/xxx, so Table A.1.1@[y8rw3hdr] means the Table A.2.1 found at http://tinyurl.com/y8rw3hdr.

2 Mathematical techniques for measuring technical efficiency

In this section, the main techniques used in this work for studying the efficiency of manufacturing firms are presented.

2.1 Multidirectional efficiency analysis

The MEA model was proposed by [4] and further developed in [5] and [2]. In what follows, we give a general description of the model and fix notation.

Let $n = (c, s, t) \in \mathcal{N}$ be a tuple identifying the firm $c \in C$, the sector $s \in S$ and year $t \in T$, which we call a firm/sector/year tuple, and $[m]$ denotes the set $\{1, ..., m\}$, for some $m \in \mathbb{N}$.

We consider that any given tuple $n \in \mathcal{N}$ produces $J \in \mathbb{N}$ outputs $y_j(n)$, $j \in [J]$, using $I \in \mathbb{N}$ inputs $x_i(n)$, $i \in [I]$, where the first $1 < D \leq I$ inputs are the so-called discretionary inputs, i.e. variables that enter into the optimization process, because the non-discretionary inputs are variables that cannot be changed. Hence, $x(n) \in \mathbb{R}^I$ is the vector of all the inputs and $y(n) \in \mathbb{R}^J$ is the vector of all the outputs, for a given sector/year tuple $n \in \mathcal{N}$. Our dataset $Z = \{z(n)\}_{n \in \mathcal{N}}$ is the set of values $z(n) = (x(n), y(n))$ for all $n \in \mathcal{N}$.

Considering the Variable Returns to Scale (VRS) model for the efficiency measurement of decision making units, see [6], we define the set

$$\Lambda^N = \left\{ \lambda \in \mathbb{R}^{\mathbb{N}} : \sum_{n=1}^{N} \lambda_n = 1 \right\}. \tag{2.1}$$

The MEA score for a specific observation $z(\bar{n}) = (x(\bar{n}), y(\bar{n}))$ is found by solving the following linear optimization programs:

$$\begin{array}{ll}
\underline{\text{Problem } P_m^\alpha(z, \bar{n})}: & \underline{\text{Problem } P_j^\beta(z, \bar{n})}: \\
\min \alpha_m(\bar{n}) \textit{ such that} & \max \beta_j(\bar{n}) \textit{ such that} \\
\sum_n \lambda_n x_m(n) \leq \alpha_m(\bar{n}), & \sum_n \lambda_n x_i(n) \leq x_i(\bar{n}),\ i \in [I], \\
\sum_n \lambda_n x_i(n) \leq x_i(\bar{n}),\ i \in [I], i \neq m, & \sum_n \lambda_n y_s(n) \geq \beta_j(\bar{n}), s \in [J], \\
\sum_n \lambda_n y_l(n) \geq y_l(\bar{n}),\ l \in [J], & \sum_n \lambda_n y_l(n) \geq y_l(\bar{n}), l \in [J], l \neq j,
\end{array} \tag{2.2}$$

$$\begin{array}{l}\underline{\text{Problem } P^{\gamma}(\alpha,\beta,z,\bar{n})}: \\ \max \gamma(\bar{n}) \textit{ such that} \\ \sum_n \lambda_n x_i(n) \leq x_i(\bar{n}) - \gamma(\bar{n})(x_i(\bar{n}) - \alpha_i^*(\bar{n})), i \in [M], \\ \sum_n \lambda_n x_i(n) \leq x_i(\bar{n}), i \in [I] \setminus \{m\}, \\ \sum_n \lambda_n y_l(n) \geq y_l(\bar{n}) + \gamma(\bar{n})(\beta_l^*(\bar{n}) - y_l(\bar{n})), l \in [L], \end{array} \tag{2.3}$$

where $\lambda \in \Lambda^n$, $\alpha_m^*(\bar{n})$ and $\beta_j^*(\bar{n})$ are the optimal solutions to the problems $P_m^{\alpha}(z,\bar{n})$ and $P_j^{\beta}(z,\bar{n})$ respectively. The ideal point of $(x(\bar{n}), y(\bar{n})$ is given by the MEA output vector

$$\zeta(n) \doteq (\alpha_1^*(n), ..., \alpha_d^*(n), ..., x_I(n), \beta_1^*(n), ..., , \beta_J^*(n)). \tag{2.4}$$

Often, some of the input variables may be discretionary (their values can be changed) and others may be not discretionary (are fixed). From now on, the discretionary variables are represented by the first indices d, $1 < d < I$. Thus, $i \in [D]$ indicates the discretionary inputs and $i \in [I] \setminus d$ indicates the non-discretionary inputs. In this setting, the MEA for a specific observation $z(\bar{n}) = (x(\bar{n}), y(\bar{n}))$ consists of $(|D| + |J| + 1) \times N$ linear programs, which includes one problem $P_d^{\alpha}(z,\bar{n})$ for each discretionary input $d \in [D]$, one problem $P_j^{\beta}(z,\bar{n})$ for each of the output dimensions $j \in [J]$ and one problem $P^{\gamma}(\alpha,\beta,z,\bar{n})$.

Definition 2.1. For a given dataset $z = \{z(n)\}_{n\in\mathcal{N}}$ the MEA score of each $n \in \mathcal{N}$ is given by

$$MEA_z(n) = \frac{\frac{1}{\gamma^*(n)} - \frac{1}{D}\sum_{i=1}^{D} \frac{x_i(n)-\alpha_i^*(n)}{x_i(n)}}{\frac{1}{\gamma^*(n)} + \frac{1}{J}\sum_{j=1}^{J} \frac{\beta_j^*(n)-y_j(n)}{y_j(n)}}, \tag{2.5}$$

where $\alpha_i^*(n)$, $\beta_j^*(n)$ and $\gamma^*(n)$ represent the corresponding optimal solutions to the linear optimization problems $P_i^{\alpha}(z,n)$, $P_j^{\beta}(z,n)$ and $P^{\gamma}(z,n,\alpha^*,\beta^*)$.

The MEA score is then obtain by the directional contribution of each input and each output variable. In fact, for the input $i \in [I]$ the contribution in the unit $z(\bar{n})$ is given by

$$\text{meff}_i(n) \doteq \frac{x_i(n) - \gamma(n)(x_i(n) - \alpha_i^*(n))}{x_i(n)} \chi_{[D]}(i), \tag{2.6}$$

where $\chi_{[D]}$ is the characteristics function of the set $[D]$. That means $\chi_{[D]}(i) = 1$, if $i \in [D]$ and $\chi_{[D]}(i) = o$ if $i \notin [D]$.

For the outputs $j \in [J]$ the contribution is given by

$$\text{meff}_j(n) \doteq \frac{y_j(n)}{y_j(n) + \gamma(n)(\beta_j^*(n) - y_j(n))}, \tag{2.7}$$

The interested reader can review in the Appendix section, a interesting remark, which allows the comparison of both the efficiency for each firm of two given variables (inputs or outputs) and the efficiency for a set of firms with respect to those variables.

One interesting feature of MEA is that the inefficiency of each input can be analyzed individually. In fact, using the ideas in [6], we calculate the following inefficiency index.

Definition 2.2. For a given dataset $z = \{z(n)\}_{n \in \mathcal{N}}$ the inefficiency index for each given input index $i \in [I]$ and tuple $n \in N$ is given by

$$R_i(n) = \frac{\sum_{n=1}^{N} \gamma(n)(x_i(n) - \alpha_i^*(n))}{\sum_{n=1}^{N} x_i(n)}, \tag{2.8}$$

We refer to the inefficiency index to determine the number of times each input was used inefficiently, since our particular interest is to assess if global efficiency can be improved with less inputs.

2.1.1 Model fitting and MEA reduction

In addition to the inefficiency index presented before, we propose a method based on model fitting by least squares to analyze the efficiency of the variables separately. Firstly, we present the following result, which allows the reduction of the dimensionality of MEA which is useful when dealing with huge databases.

Lemma 1 Let $z = \{x(n), y(n)\}_{n \in \mathcal{N}}$ be a firm dataset with associated MEA output vector $\zeta(n)$. Suppose there exist nonempty subsets $C_I \subset I$, $C_J \subset J$ and sobrejective maps $\varphi : I \to C_I$ and $\psi : J \to C_J$ such that

$$\begin{aligned} x_i(n) \equiv x_i(c,s,t) &\approx a_i(t) x_{\varphi(i)}(n) + b_i(t), \\ y_j(n) \equiv y_j(c,s,t) &\approx c_j(t) y_{\psi_{(j)}}(n) + d_j(t) \end{aligned} \tag{2.9}$$

for all $i \in [I]$ with p-value $p_i(t)$ and all $j \in [J]$ with p-value $q_j(t)$, where $a_i(t)$, $b_i(t)$, $c_j(t)$, $d_j(t) \in \mathbb{R}$ and $a_i(t), c_j(t) > 0$. If we consider $\sum_{n=1}^{N} \lambda_n = 1$ in the optimization MEA problems, then

$$\begin{aligned} \alpha_i^*(n) &= a_i(t) \alpha_{\varphi(i)}^*(n) + b_i(t), \\ \beta_j^*(n) &= c_j(t) \beta_{\psi(j)}^*(n) + d_j(t), \end{aligned} \tag{2.10}$$

and the score for $\tilde{z}(n) = (x_{\varphi(i)}(n), y_{\psi_{(j)}}(n))$ is given by

$$MEA_{\tilde{z}}(n) = \frac{\frac{1}{\gamma(n)} - 1 + \frac{1}{D} \sum_{i=1}^{D} \frac{a_i(t)\alpha_{\varphi(i)}^*(n) + b_i(t)}{a_i(t) x_{\varphi(i)}(n) + b_i(t)}}{\frac{1}{\gamma(n)} - 1 + \frac{1}{J} \sum_{j=1}^{J} \frac{c_j(t)\beta_{\psi(j)}^*(n) + d_j(t)}{c_j(t) x_{\psi(j)}(n) + d_j(t)}}. \tag{2.11}$$

We refer the interested readers to see the proof of this result in the Appendix section.

The procedure based on model fitting is the following. We choose functions φ and ψ, by computing the coefficient functions which produce the best p-values (i.e. the so-called calculated probability or probability value). The coefficients $a_i(t)$ and $c_j(t)$ generated by model fitting represent the coefficients of the relationship between each variable with $x_{\varphi(i)}$ and $y_{\psi(j)}$, respectively. The terms $b_i(t)$ and $d_j(t)$ are the intercepts of these relations.

2.1.2 Cluster analysis

A standard and natural requirement is to study the performance of firms according to their sizes. For such it is common to use pre-established rules for the group creation, for example, according to the European Commission (recommendation 2003/361/EC), firms are classified as micro, small, medium or large using the following criteria. If a firm employs fewer than 250 persons and have either an annual turnover not exceeding 50 million euros or an annual balance sheet total not exceeding 43 million euros, it is included in the group of SMEs (micro, small and medium enterprises). Within this group, small firms are the ones which employ fewer than 50 persons and whose annual turnover or annual balance sheet total does not exceed 10 million euros. Finally, micro enterprises are defined as firms which employ fewer than 10 persons and whose annual turnover or annual balance sheet total does not exceed 2 million euros. A problem with the use of these, or similar, criteria when comparing performances along time is that a firm can move from one group to another by just increasing one employee, so the classification may not be stable along the years of study.

In this work, we decided to use a clustering algorithm [11] to perform the division of firms by size-based groups compatible with the given data features. Among many algorithms for performing non-hierarchical clustering, the Partitioning Around Medoids (PAM) proposed in [16, 17] is known to be the most powerful one of these situations, see [15]. The PAM algorithm first computes k medoids (representative objects of a cluster whose average dissimilarity to all the objects in the cluster is minimal). After finding the set of k medoids, each object of the data set is assigned to the nearest medoid. This algorithm works with a matrix of dissimilarity, whose goal is to minimize the overall dissimilarity between the representants of each cluster and its members. Let $X = (x_1, x_2, ..., x_n)$ be a set of objects and $d(i,j)$ the dissimilarity between two objects $x_i, x_j \in X$. The PAM algorithm solve the following problem:

$$C(x) = \min \sum_{i=1}^{n} \sum_{j=1}^{n} d(i,j) z_{ij} \tag{2.12}$$

such that

$$\begin{aligned}
&\textstyle\sum_{i=1}^{n} z_{ij} = 1, j = 1, 2, ..., n\\
&z_{ij} \leq y_i, j = 1, 2, ..., n\\
&\textstyle\sum_{i=1}^{n} y_i = k, \text{k=number of clusters}\\
&y_i, z_{ij} \in \{0, 1\}, j = 1, 2, ..., n
\end{aligned} \tag{2.13}$$

where the variable z_{ij} ensures that only the dissimilarity between entities from the same cluster will be computed in the main function.

Compared with other algorithms, PAM is quite robust in the sense that it minimizes a sum of dissimilarities instead of a sum of squared Euclidean distances. In this paper, we use the PAM algorithm based on the GDM2 distance measure proposed in [24].

2.1.3 Principal component analysis with a test of dimensionality

To select the most meaningful input and output variables, we use Principal Component Analysis (PCA). This multivariate technique proposed in [19] transforms a number of correlated variables into a (smaller) number of uncorrelated variables. The PCA involves the calculation of the eigenvalue decomposition of a data covariance matrix or singular value decomposition of a data matrix, usually after mean centering the data for each attribute. In order to avoid loss of information (underfitting) or the introduction of random noise (overfitting), we perform the test of dimensionality so-called *testdim* [9], which allow to test for the number of axes in multivariate analysis. The procedure is based on the computation of the RV coefficient, introduced in [21]. We briefly explain it in what follows.

Let X be a table with the measurements of p centered variables (columns) for n units (rows) and set the singular value decomposition of $X^* = (1/\sqrt{n})\,X = UDV^t$, where D is a diagonal matrix $(r \times r)$ with the r non-null singular values $D = diag(d_1, d_2, \ldots, d_r)$ sorted in decreasing order $(d_1 > d_2 > \cdots > d_r > 0)$. The column vectors in $U = [u_1|\cdots|u_r]$, of size $n \times r$, and $V = [v_1|\cdots|v_r]$, of size $p \times r$, are orthonormal and verify $U^tU = V^tV = I_r$. Considering the best approximation of X in the sense of least squares, we have $X = \sum_{j=1}^{i} X_j + R_{i+1}$, where $X_j = d_j u_j v_j^t$ and R_i represents the residuals [13]. We need to know if an element X_i adds relevant information to the decomposition $\hat{X}_{i-1}$ of rank $i-1$. The test proposed is based on the similarity between X_i and R_i. The RV coefficient is a measurement of the closeness between the configuration to the representation of the units in the unidimensional space formed by the i^{th} principal axis, and the configuration of individuals in the $(r-i+1)$-dimensional space formed by the last $(r-i+1)$ principal axes [21]. In [9] is proposed a corresponding dimensional RV statistic defined by

$$DIM_{RV}(X_i,R_i) = \frac{\operatorname{tr}\left(X_i^t R_i R_i^t X_i\right)}{\sqrt{\operatorname{tr}\left(X_i^t X_i X_i^t X_i\right)\operatorname{tr}\left(R_i^t R_i R_i^t R_i\right)}} = \frac{\lambda_i}{\sqrt{\sum_{j=i}^{r}\lambda_j^2}}.$$

Then our variable selection is made by evaluating the p-values for the i^{th} axis X_i, details of the implementation of a randomized algorithm can be found in [9].

2.1.4 The NC-value

In order, to compare the behaviour of input and output variables between two groups F_1 and F_2 with different levels of efficiency, we use the NC-value proposed by [14] for measuring the overlapping of Gaussian distribution functions. This procedure requires that we define which are the firms in each group. The variable to be studied (e.g. efficiency score) in each group gives a mean and a standard deviation that generates a Normal distribution. So, allowing to compute the NC-value between the Normal distributions of the groups. Such is done for each sub-dataset, variable and year of interest.

Let μ_1^t, μ_2^t be the average and σ_1^t, σ_2^t be the standard desviation of the groups F_1 and F_2 in the time $t \in T$, respectively. Assume $\mu_1^t < \mu_2^t$, without lost of generality. If $X_1 \sim N(\mu_1,\sigma_1)$ and $X_2 \sim N(\mu_2,\sigma_2)$ then

$$\begin{aligned} NC-value &= \frac{1}{|T|}\sum_{t\in T} P(X_1 > c^t) + P(X_2 < c^t) \\ &= \frac{1}{|T|}\sum_{t\in T}\left(1 - \frac{1}{2}\operatorname{erf}\left(\frac{c^t-\mu_1^t}{\sqrt{2}\sigma_1^t}\right) + \frac{1}{2}\operatorname{erf}\left(\frac{c^t-\mu_2^t}{\sqrt{2}\sigma_2^t}\right)\right), \end{aligned} \tag{2.14}$$

where the point c^t is calculated as

$$c^t = \frac{\mu_2^t(\sigma_1^t)^2 - \sigma_2^t\left(\mu_1^t(\sigma_2^t) + \sigma_1^t\sqrt{(\mu_1^t-\mu_2^t)^2 + 2\left((\sigma_1^t)^2-(\sigma_2^t)^2\right)\log\left(\frac{\sigma_1^t}{\sigma_2^t}\right)}\right)}{\left(\sigma_1^t\right)^2 - \left(\sigma_2^t\right)^2}. \tag{2.15}$$

The smaller the NC-value, the less common the behaviour of the two groups with respect to the selected variables.

3 Empirical Application

3.1 Contextualization and Hypotheses

Many of the larger Portuguese firms focus their activity on sectors whose competitiveness is based on natural and environmental resources with the internal market as a growth horizon, such includes areas such as construction and infrastructure sectors, the forest industry (wood and pellets, cork and pellets, pulp and paper) and the agricultural industries (wines, cooking oil). However, for increased export capacity, the Portuguese Government's commitment is based on sectors such as the fashion industries (textiles, clothing and footwear); food processing (wine, fruit and vegetables or flowers); mobility industries (automotive and aerospace); health services; molds and tools industry; information technology, communications and electronics; equipment industries/production technologies, particularly in the environmental and refining, petrochemical and industrial chemistry.

In most industries, some factors negatively influence the strategic performance of their firms: the weak financial capacity, poor preparation of human resources and the high dependence on external raw materials. In the manufacturing sector, more than in any other sector, the challenges are constant, markets change and technologies evolve. Although in recent years, Portuguese firms have been faced with major challenges, the major goals continue to be to create strategies to improve the time-to-market, increase quality, and efficiently manage their headquarters. When it comes to the type of investment carried out by the manufacturing sector, it turns out that small and medium enterprises currently make a targeted effort to adapt to quality and environmental protection standards while in all major firms, the investment is already channeled into areas such as cogeneration or innovation and some even face up to the challenge of internationalization. Products such as the ones from food and beverage industries are of considerable importance in the calculation of the Portuguese foreign trade.

After the international economic crisis, the Portuguese economy was in a phase of structural adjustment. The Portuguese business dynamics have undergone major changes during the first two years of the bailout, 2011 and 2012, verifying an increase of closures and an escalation in the number of insolvencies, which reached the highest value since 2007. However for some manufacturing firms the degree of reaction to the crisis is not necessarily negatively related to productivity. These firms show greater flexibility over other enterprises. Here flexibility refers to the organizational capability to adapt to changes either within the firm or in its environment. In this sense it might be thought that there are subsectors that have become more efficient in spite of the crises, somehow improving its financial management.

In our study, we have used a set of Portuguese industrial firms observed between 2006 and 2013. The data were divided into seven sub-sectors (described in detail in the Section 3.2), which in turn are divided into two groups: sectors with a small number of firms (group A) and sectors with a large number of firms (group B). As mentioned above, the purpose of this study is to examine the effects of the financial crisis on the manufacturing sector of Portuguese industry efficiency in relation to their effects in three distinct phases: the levels of efficiency; efficiency standards; and determinants of efficiency. To this end, our study is motivated on three aspects:

1. The changes in each of the Portuguese firms in all economic sectors after the crisis seem obvious, but in the end, a question hangs in the air: to what extent has the crisis affected negatively the performance of the firms? According to studies by the Portuguese National Statistics Institute, it has been reported that industrial production in Portugal had an average decreasing of -0,63 %, from 2005 to 2015. In fact, it has been highly oscillatory during such period, reaching a peak of 13,40 %, in April 2011, and a low of 50%, in January 2009. Therefore, we want to measure in which the extent the economic crisis was detrimental or favorable to the manufacturing sector.

2. As it is well known, many firms in Europe did not manage to confront the challenges generated by the crisis and were closed. To deal with these, the acquisition of debt long term was one of the most common outputs assumed by the resistant firms to overcome the difficulties caused by the crisis (drop in sales, decline in efficiency, impairment in economically-financial management). We want to know, what was the attitude of the manufacturing sector in front to the acquisition of the debt and whether this alternative solution, lead effectively to an improvement of the efficiency of their firms or by contrary, even with huge debts acquired during the pre-troika and troika period, the manufacturing sector, failed to improve its solvency.

3. Study the behavior of the food subsector under the crisis, has a special interest, since based on information from the Central Balance-sheet Database of Banco de Portugal (period 2006-2009) (see [3]), notwithstanding a decline in 2009, manufacture of food products financing through trade credits grew by 9% in the 2006-2009 period. In 2009 the food products sector represented approximately 14% of the number of firms, 13% of the number of employees and 16% of turnover in manufacturing. Compared to the overall results of international trade, manufacturing of food products was responsible for more than 6% of national exports of goods.

To analyze the three aspects mentioned above, our study focused on three main hypotheses:

H(1): The performance of the portuguese firms in the manufacturing sector has been adversely affected by the financial crisis felt in Portugal in the troika years;
H(2): Due to the financial crisis, the manufacturing sector acquired long-term debt deliberately;
H(3): The financial crisis has affected substantially the food subsector.

3.2 Characterization of the data

Our data set, collected from the Amadeus (Bureau van Dijk) database, comprises financial information and other characteristics of Portuguese firms operating in the manufacturing sector (NACE Rev.2-Statistical classification of economic activities in the European Community, section C, codes between 1000 and 3399). We considered only the firms with data available for all years in the period 2006-2013 and partitioned the manufacturing sector in seven subgroups as follows (NACE codes in parentheses):

1. FOOD $(1000-1299)$: Manufacture of food products, beverages and tobacco products - 553 firms;

2. TEXT $(1300-1599)$: Manufacture of textiles, apparel, leather and related products - 727 firms;

3. MATER $(1600-1899, 2200-2599)$: Manufacture of wood and paper products, and printing; Manufacture of rubber and plastics products, and other non-metallic mineral products; Manufacture of basic metals and fabricated metal products, except machinery and equipment - 1787 firms;

4. CHEM $(1900-2099)$: Manufacture of coke, and refined petroleum products; Manufacture of chemicals and chemical product - 103 firms;

5. MED $(2100-2199)$: Manufacture of pharmaceuticals, medicinal chemical and botanical products - 21 firms;

6. EQUIP $(2600-2899, 3100-3399)$: Manufacture of computer, electronic and optical products; Manufacture of electrical equipment; Manufacture of machinery and equipment n.e.c; Other manufacturing, and repair and installation of machinery and equipment - 692 firms;

7. TRANSP $(2900-3099)$: Manufacture of transport equipment - 94 firms.

Considering our database, we analyzed the efficiency of 23.862 units (3.977 firms in six years). For each year and firm, we extracted from the Amadeus database the following 13 variables: (1) number of employees (NE); (2) cash and cash equivalent (CASH); (3) issued share capital (CAPITAL); (4) total assets (TASSETS); (5) long term debt (LTDEBT), defined as the company's total debt due for repayment beyond one year; (6) profit margin (PROFITM), found by multiplying by 100 the result of dividing profit before tax by operating revenue; (7) current liabilities (CLIAB); (8) liquidity ratio (LIQR), defined by the difference between current assets and stocks, divided by current liabilities; (9) solvency ratio (SOLVR), calculated by multiplying by 100 the result of dividing the shareholders funds and total assets; (10) sales (SALES); (11) EBIT margin (EBITM), calculated by multiplying by 100 the result of dividing the difference between all operating revenues and all operating expenses by operating revenue; (12) EBITDA margin (EBITDAM), found by multiplying by

100 the result of dividing the sum of operating profit and depreciation by operating revenue; and (13) cash flow (CASHFLOW).

The methodology to test the hypotheses, formulated in the previous section, follows the structure presented in Fig 1.

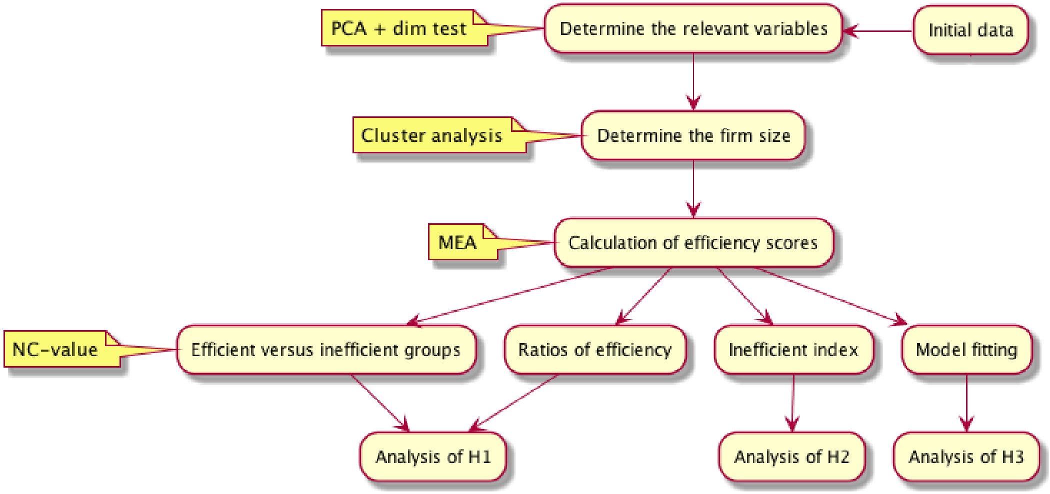


Figure 1: Structure of the applied methodology in order to answer the hypotheses.

3.3 Hypotheses Analysis and Results

This section uses MEA to study the effects of the financial crisis on the efficiency of the Portuguese manufacturing sector. First, a brief description of the methodology used in the analysis is provided. Then, the main results obtained are reported and commented on. We found that the best criteria for clustering the firms is obtained when using the variables NE and SALES. For our data, the optimal number of clusters is given by only 2 clusters: (cluster 1) corresponding to firms with a mean(NE) around 100 and std(NE) around 148,3; and (cluster 2) with a mean(NE) around 10 and std(NE) around 7,2. Recall that the method applied insures the best stability of the clusters along the eight years. From applying PCA and the dimensionality test, we obtain that the variables that should be considered in the estimation of efficiency of manufacturing sector firms are the inputs: NE, TASSETS, LTDEBT, CLIAB and the outputs: PROFITM, LIQR, SOLVR, EBITM, EBITDAM. Descriptive statistics for these four inputs and five outputs according to the two clusters in the food sector are exhibited in Tables A.1.1-A.1.4@[y8rw3hdr], respectively.

We apply MEA model to the nine variables selected in all subsectors. Define EFF as the subset of tuples $n = (c, s, t)$ such that $0,6 \leq MEA_Z(n) \leq 1,0$ for a fixed sector $s \in S$, where $MEA_Z(n)$ denotes the MEA score (see Equation 2.5). As

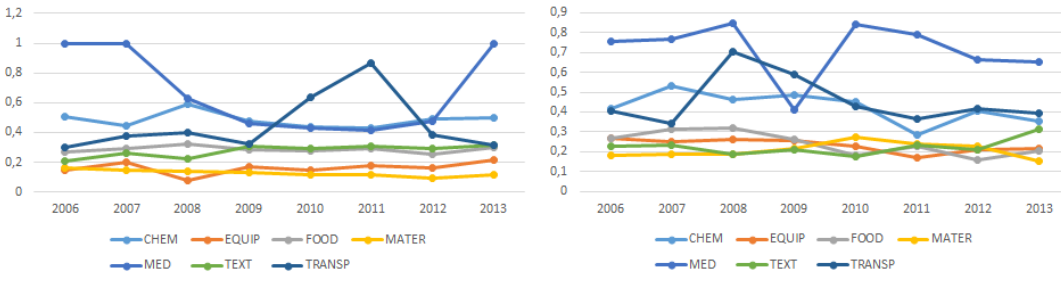


Figure 2: (a) EFF mean, cluster 1; (b) EFF mean, cluster 2 (on the right).

we can see in Fig 2, CHEM, MED and TRANSP subsectors are consistently more efficient than the others subsectors along the years. Meanwhile, the least efficient subsectors vary depending on the cluster. Nevertheless, the MATER always remains as one of the least efficient subsectors regardless of the division. The results on the figures are as expected, since we consider subsectors with large differences in size. For instance the MED subsector is in a higher percentage of efficiency and MATER in a smaller percentage, being one of the reasons the fact that MED has only 21 companies and MATER 1787. Therefore, it becomes natural to divide our study into 2 groups. Group S_1: MED, TRANSP and CHEM; and group S_2: FOOD, EQUIP, TEXT and MATER. The intention with Fig 2 is to show how efficiencies are distributed through the years and how the clusters influence in both groups.

3.3.1 About the hypothesis H(1).

For analyzing the performance of the firms during the troika years, we pose two approaches.

We first examine the different subsectors and try to establish differences in their levels of efficiency in the last two periods. Analyzing the resulting ratios efficiencies, we noticed that the efficiency of the firms decreased after the crisis for most subsectors. Over the troika period a decrease in efficiency ratios between $26,36\%$ and $4,15\%$ for cluster 1 and between $0,65\%$ and $8,81\%$ was presented for cluster 2. However, two of the seven subsectors surveyed improve efficiency, which is the case of TEXT that increased $2,47\%$ in cluster 1 and $0,92\%$ in the cluster 2. For TRANSP the increase was much higher of $6,6\%$ in the cluster 1 and $10,5\%$ in cluster 2. To determine in detail the changes of this subsector compared to CHEM, a summary of the resulting ratios efficiencies is provided in the Table 1. In further detail, we may extract three relevant quantities from the efficiency scores. Fixed $s \in [S]$

and $t \in [T]$, we define: **efficient firms** (EFF) as the percentage of tuples n such $0,6 \leq MEA_Z(n) \leq 1,0$; **fully efficient firms** (FullEFF) as the percentage of firms with efficiency equal to 1; and **null efficient firms** (NullEFF) as the percentage of firms with efficiency equal to 0. The year 2008 is a representative year (see Table 1). In fact, EFF has the highest percentage in TRANSP and the lowest percentage in CHEM. On the other hand, in TRANSP, 2009 was characterized by trend where the percentage of FullEFF is lower than NullEFF; contrary to the CHEM which in all the years the percentages in FullEFF are higher than NullEFF. However, a real change in the efficiency of TRANSP occurred in the year 2010 with a critical decline in 2011. Although when there is an improvement in the efficiency, over troika period, it never resembles the pre-crisis period.

	CHEM			TRANSP		
Year	EFF	FullEFF	NullEFF	EFF	FullEFF	NullEFF
2006	72,0	28,0	28,0	70,8	25,0	29,2
2007	80,0	28,0	20,0	75,0	25,0	25,0
2008	68,0	32,0	32,0	75,0	29,2	25,0
2009	74,0	26,0	26,0	72,9	20,8	27,1
2010	92,0	28,0	8,0	20,8	12,5	79,2
2011	92,0	44,0	8,0	14,6	12,5	85,4
2012	78,0	30,0	22,0	33,3	12,5	66,7
2013	82,0	32,0	18,0	52,1	14,6	47,9

Table 1: Efficiency ratios statistics of CHEM and TRANSP in the cluster 1.

Now, we verify the hypothesis H(1), by confronting the increase or decrease of the number of employees of the firms in the last two periods. To this end, we calculate the NC-value for each variable (see Section 2.1.4), considering two groups in each subsector. So, let G_1 be composed by the more efficient firms, corresponding to the units such that $0,6 \leq$ MEA score $\leq 1,0$; and G_0 be composed by the less efficient firms, corresponding to the units such that $0,0 \leq$ MEA score $\leq 0,4$. As a result of the crisis, in the 64% of firms (on the groups in the majority belonging to the sub-sectors CHEM, EQUIP and MATER) the number of employees lowered, during the period troika. However, in some subsectors with the highest number of firms such as TEXT and FOOD managed to increase the number of employees in the same period. The sectors with fewer firms, as TRANSP and MED, showed different behavior for firms in G_0 and G_1. See Fig 3, which represent the behaviour of the more efficient set of firms G_1 and less efficient set of firms G_0, for NE in TEXT and FOOD, respectively. In the first case the NC-value is equal to 305 and the second case the NC-value is $136,1$. TRANSP contrary to MED increased the NE in the more efficient companies, during the period troika and lowered in the less efficient. On the other hand, in Food subsector, contrary to what was expected,

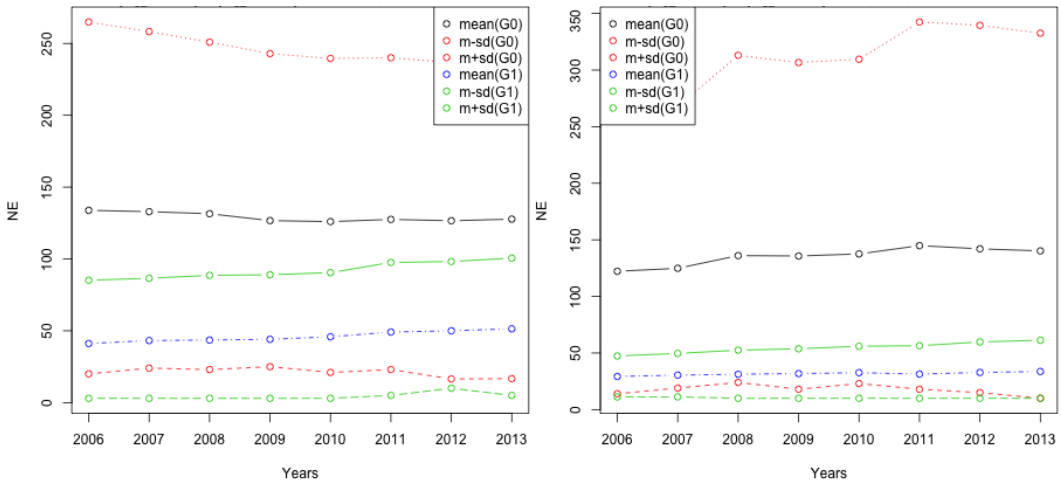


Figure 3: NC-value for the variable NE in cluster 1: (a) TEXT; (b) FOOD (on the right).

for both clusters, during the pre-troika and troika periods, the number of employees was increased (see Table A.1.1-A.1.2@[y8rw3hdr]).

To conclude, examining the seven subsectors under the two approaches presented above, we note that the crisis affected the performance of the firms. Nevertheless, contrary to what could be expected, there is not exactly a "negative" influence of the crises in all the subsectors under study, inversely, there is a broad adaptability of some companies towards it.

3.3.2 About the hypothesis H(2).

The hypothesis H(2) translates to analyze whether during the pre-troika and troika periods, LTDEBT was the variable more used inefficiently in all subsectors. To this end, we use the inefficiency index (Equation 2.8). Recall, that the percentages represent the number of times each input was used inefficiently (excess inputs). Detailed results of the inefficiency index for sector group S_1 with respect to cluster 1 are presented in the Table 2. As we can see, during the pre-troika period, the three subsectors showed very different behaviours. In CHEM the variable that is used with less inefficiency is NE, and in MED is TASSETS. While in TRANSP changes in each year and it is difficult to establish which is the variable best used. In 2010, LDEBT was the variable more used inefficiently in all subsectors, so we may assume that this year it was increased the acquisitions of debt. However, in TRANSP, the percentage is not as high as in the other two subsectors. If we consider

YEAR	Sector	A	B	C	D	Sector	A	B	C	D
2009	CHEM	76,7	95,0	98,5	94,3	MED	70,8	42,6	67,9	57,5
2010		72,0	95,2	95,5	94,7		71,7	62,3	95,8	68,0
2011		66,7	93,3	91,8	93,8		66,2	54,9	53,8	72,6
2009	TRANSP	57,1	67,9	74,4	81,7	TEXT	66,1	69,0	73,8	75,2
2010		80,2	63,8	84,9	67,6		71,0	78,5	87,6	79,3
2011		83,0	84,9	95,1	92,8		67,2	75,5	85,2	77,8

Table 2: Inefficiency index of some subsectors for the inputs: A (NE), B (TASSETS), C (LTDEBT) and D (CLIAB).

the accumulate inefficiency index of all variables of a sector by year, we may verify, globally in the period 2009-2011, that CHEM decreased the inefficient use of inputs, MED and TEXT have a peek in 2009, but recovered in 2011, and TRANSP steadily increased the inefficiency, being the worst behaved subsector with regard to input usage.

Henceforth, we are interested in analyzing the changes suffered during the crises by the subsectors more efficient of each sector group, to determine if the efficiency patterns present similarities between these subsectors, regarding the acquisition of debt. Note that in Fig 2, TRANSP and TEXT are the most efficient subsectors of sectors group S_1 and S_2 respectively, during the crisis (time period 2006-2009). There are some similarities between these two subsectors. LTDEBT is the variable most used inefficiently and NE is the least used (see Table 2). There is an interesting relationship between the percentage of LTDEBT to be used inefficiently and the ratio of efficiency EFF, in the sense that there is some trend showing a proportional relation between the LTDEBT inefficiency and the EFF value. A pertinent observation showed that less efficient firms borrowed more during the crisis but failed to improve its liquidity in the years immediately following. Contrary, more efficient firms did not assume attitudes that ultimately negatively affect their performance, these firms did not resort to big debts. See, for instance, the behaviour of the more efficient set of firms G_1 and less efficient set of firms G_0, in TEXT, for LTDEBT and LIQR, represented in Fig 4(a) and Fig 4(b) respectively. In this subsector, G_0 has 226 firms and G_1 has 24 firms. In LTDEBT, NC-value is $221,9$ (Fig 4(a)); and in LIQR, NC-value is $426,5$ (Fig 4(b)).

In Table 3, we present the NC-value for each variable in each sector, corresponding to the cluster 1. The Figures B.1.1.1-B.1.7.13@[ybq2pw07] and Figures B.2.1.1-B.2.7.13@[y9wxehe9], represent the behaviour of the more efficient and the less efficient set of firms for each variable in each subsector; in the cluster 1 and in the cluster 2, respectively. Recall that a higher NC-value means greater intersection between the behavior of the groups G_0 and G_1.

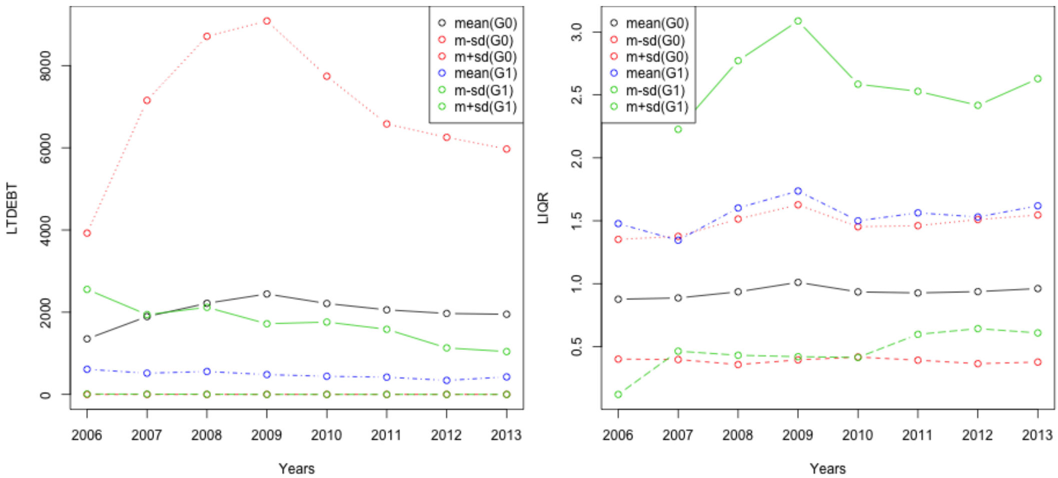


Figure 4: NC-value in the TEXT, for cluster 1: (a) LTDEBT; (b) LIQR.

NC-value	CHEM	EQUIP	FOOD	MATER	MED	TEXT	TRANSP
PROFITM	46,1	67,4	37,6	1,2	213,6	63,	130,5
CLIAB	14,5	415,1	67,4	100,1	231,6	155,3	357,4
LTDEBT	44,1	211,2	71,2	117,8	30,27	221,9	314,7
SALES	13,9	221,8	97,2	181,9	394,4	341,6	523,6
TASSETS	20,2	192,0	97,8	235,3	483,8	257,1	548,7
EBITM	43,2	71,6	125,4	10,2	242,9	267,0	171,9
NE	67,2	268,2	136,1	197,8	210,7	305,0	472,3
CASHFLOW	73,9	429,7	169,4	486,6	372,9	435,9	432,4
CAPITAL	15,0	365,5	182,5	442,9	213,2	1148,5	143,3
CASH	12,1	255,7	235,1	550,4	89,5	290,3	66,4
SOLVR	27,7	176,5	266,6	70,5	103,0	359,6	155,9
LIQR	46,1	250,2	330,8	190,6	161,2	426,5	298,7
EBITDAM	39,8	222,3	379,5	93,7	297,5	152,0	271,3

Table 3: NC-value in the cluster 1.

3.3.3 About the hypothesis H(3).

To test the hypotheses H(3), we will study the relation between four key variables: CAPITAL, LTDEBT, SOLVR and EBITM. The reason for choosing these four variables is because we want to address the performance of food Portuguese companies, concerning to acquisition of debt to assess its efficiency before, during and after the crisis; SOLVR refers to the ability of the company to pay or cover their debts or obligations; the ratio EBITM indicates the ability of a company to be profitable, and ultimately to generate profits. Since CAPITAL and EBITM are relatively stable over all the years (see the behaviour of CAPITAL and EBITM in Figure C.1@[yajwqsdv] and Figure C.2@[yajwqsdv] respectively), from now on, we focus our attention on these two variables. In fact, using a standard fitting procedure for affine functions and checking the test of hypothesis (calculating p-values for the fittings), we verified that surprisingly good, p-values are obtained when relating the other variables with one of the variables CAPITAL or EBITM. In the CAPITAL case: CLIAB, LTDEBT, NE, SALES, TASSETS and CASHFLOW. In the EBITM case: LIQR, EBITDAM, PROFITM, SOLVR and CASH.

Suppose, there exist $x_i(n)$, $i \in [7]$ and $y_j(n)$, $j \in [5]$, such that

$$x_i(n) \approx a_i(t)C_i(n) + b_i(t), \quad y_j(n) \approx c_j(t)E_j(n) + d_j(t), \tag{3.1}$$

where $C_i(n)$ and $E_j(n)$ are the value of CAPITAL and EBITM respectively, $n = (c, s, t)$ and s is fixed to the FOOD subsector. The values $a_i(t)$ and $c_j(t)$ represent the coefficients of the relationship between each variable with CAPITAL and EBITM, respectively. The values $b_i(t)$ and $d_j(t)$ are the intercepts of these relations. We

Year	$a_i(t); b_i(t)$; p-ME	$c_j(t); d_j(t)$; p-ME	Year	$a_i(t); b_i(t)$; p-ME	$c_j(t); d_j(t)$; p-ME
2006	0,44; 834,63; -49	0,58; 28,06; -14	2010	0,94; 841,30; -24	0,74; 31,65; -11
2007	0,65; 1032,69; -41	0,53; 28,14; -11	2011	1,11; 457,45; -48	0,87; 32,54; -15
2008	1,23; 572,40; -35	0,50; 28,59; -7	2012	1,10; 412,87; -58	0,93; 32,05; -21
2009	1,15; 685,19; -30	0,56; 30,37; -9	2013	1,07; 391,00; -50	1,12; 30,88; -26

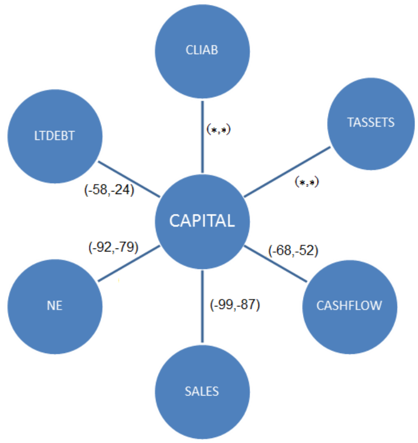
Table 4: The values $a_i(t)$, $b_i(t)$ and p-values of the relation between CAPITAL with LTDEBT; and the values $c_j(t), d_j(t)$ and p-ME of the relation between EBITM and SOLVR, in the food subsector.

represent in the Table 4, the values $a_i(t)$, $b_i(t)$; and $c_j(t)$, $d_j(t)$ for LTDEBT and SOLVR respectively, in the FOOD for the cluster 1. There, the values p-ME, are also shown, where p-ME represent the value (maximum exponent) such that p-value $\leq 10^{\text{p-ME}}$. We show, in the Table 5, the minimum and maximum values for $a_i(t)$,

Variables	A	B	Variables	A	B
CASHFLOW	[0,28; 0,80]	[-297,24; 166,26]	TASSETS	[5,04; 6,09]	[630,34; 2886,95]
CLIAB	[1,97; 3,20]	[-210,43; 1309,12]	SOLVR	[0,50; 1,12]	[165,50; 278,84]
LTDEBT	[0,44; 1,23]	[391,00; 1032,69]	EBITDAM	[0,79; 1,02]	[5,45; 7,75]
NE	[0,01; 0,02]	[28,76; 30,56]	PROFITM	[0,95; 1,03]	[-2,78; -1,58]
SALES	[4,67; 5,50]	[2415,53; 3506,64]			

Table 5: The minimum and maximum values for $a_i(t)$ or $c_j(t)$ (A); and $b_i(t)$ or $d_j(t)$ (B), respectively.

$b_i(t)$; $c_j(t)$ and $d_j(t)$ for each variable. To measure how well the fittings are along



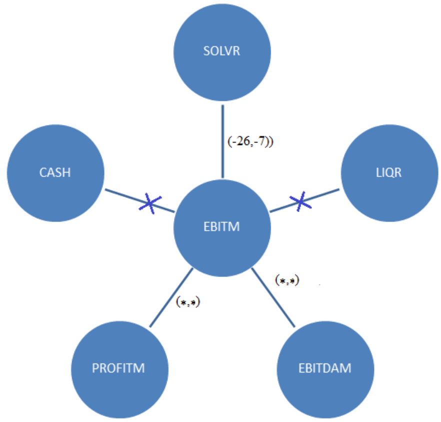


Figure 5: (a) Variables related with CAPITAL; (b) Variables related with EBITM. Here, asterisk means that $\log_{10}(p) \leq -100$ in every year. The x on the lines means that there is no acceptable fitting between the two variables.

the years, between inputs and CAPITAL; and outputs and EBITM, we present the range of p-ME values in Fig 5. If we do not consider CASH and LIQR, the fitting presented above reduces the problem to just one input and one output. Hence, not even Lemma 1 is needed here, since do not make sense to apply MEA in this context. Nevertheless, the fitting process gives a good approximation to study the problem. In what follows, we develop some new visualization techniques to extract information from the data that otherwise would be rather difficult.

In order to complete the analysis of the hypothesis H(3), we need to understand the general behavior of CAPITAL, LTDEBT, SOLVR and EBITM, on the food subsector, and their changes along the years. However, we are dealing with many firms so it is not feasible to study them individually. Further, basic statistics as means and standard deviations are not useful here. We then use the fitting information of Table 4. We first find the intersection points of the fitting lines (see Fig 6(a)), and from them, we construct a so called (first) relative order graph (see Fig 6(b)). The latter graph shows the intersection points of CAPITAL, on the x-axis, and the number, on the y-axis, indicates the order in which the fitting lines appear in that interval when y increases. The (second) relative order graph (see Fig 7) is obtained using the information in the (first) relative order graph in the following way. Choose a partition of the values on the x-axis of Fig 6(b), corresponding to CAPITAL intersection points, by dividing them in N intervals $I_i = [a_{i-1}, a_i)$ with $i \in [N]$ and

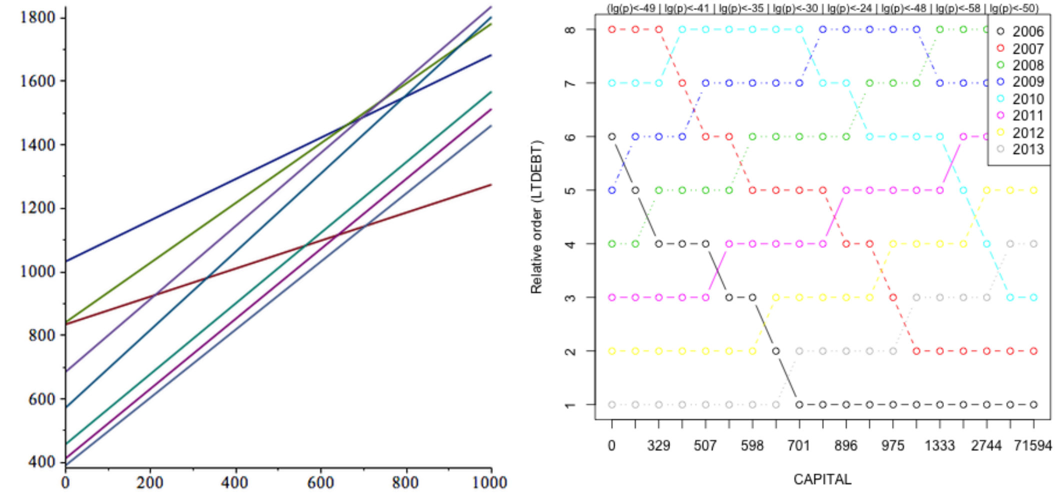


Figure 6: CAPITAL vs LTDEBT on FOOD: (a) fitting lines graph; (b) first relative order graph.

$a_0 < a_1 < ... < a_N$. Set $k(t,x) \in \mathbb{N}$ as the order of the year t for the CAPITAL intersection value x; and P_i the set of intersection points x in the interval I_i. Now, define the function

$$O_i(t) \doteq \frac{1}{|P_i|} \sum_{x \in P_i} k(t,x). \tag{3.2}$$

The plots of the functions O_i are represented in Fig 7(a), for CAPITAL vs LTDEBT. In the same way, Fig 7(b) shows O_i for EBITM vs SOLVR. For a better description of the (second) relative order graphs, we found as optimal values for the N intervals: three for CAPITAL and four for EBITM. The values on the legends show the percentage of firms in the corresponding interval. Recall that CAPITAL and EBITM are the most stable variables along the years. For space reasons, the reader can see the behaviour of CAPITAL with other variables in Figures C.3.1-C.3.7@[yajwqsdv]; and the behaviour of EBITM with other variables in Figures C.4.1-C.4.5@[yajwqsdv]. An interesting observation from the (second) relative order graphs is the diversity of regression types between the level of debt acquisition and the size of capital, which do not happen between EBIT and the solvability ratio since they are far more similar along all the range of values of EBIT. In one way, we may say that firms have been aware of the possibility of acquiring long-term debt as a solution to the crisis, with a possible positive outcome for most of the firms. In fact, they increased significantly during the troika and, as expected in later years, this level had decreased considerably, since commitments had already been made in

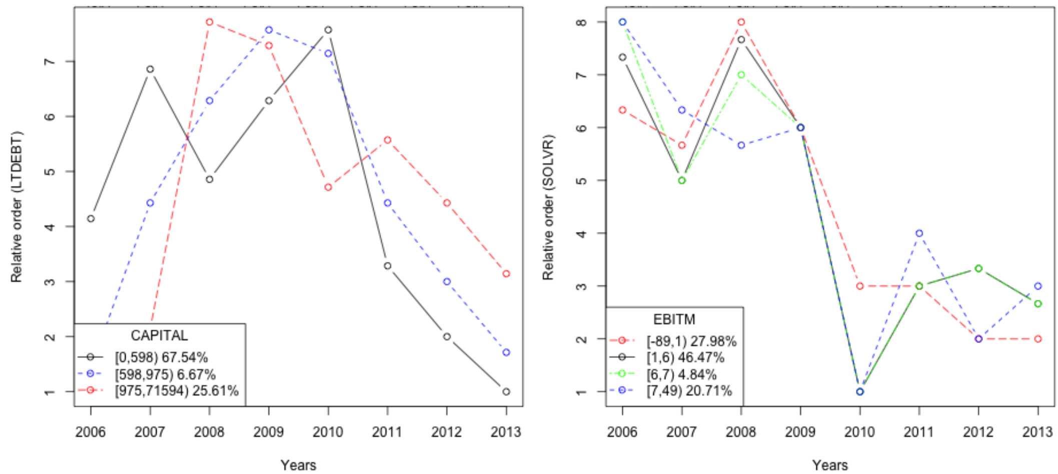


Figure 7: In the FOOD subsector: (a) CAPITAL related with LTDEBT; (b) EBITM related with SOLVR.

the long-term. On the other hand, the solvability relative order decreased in the latter years. So, it suggests that the deliberated acquisition of debt helped the lower capital firms, although reduced the overall solvency and the ability of firms to meet its long-term financial obligations. From the first point of view, the results show

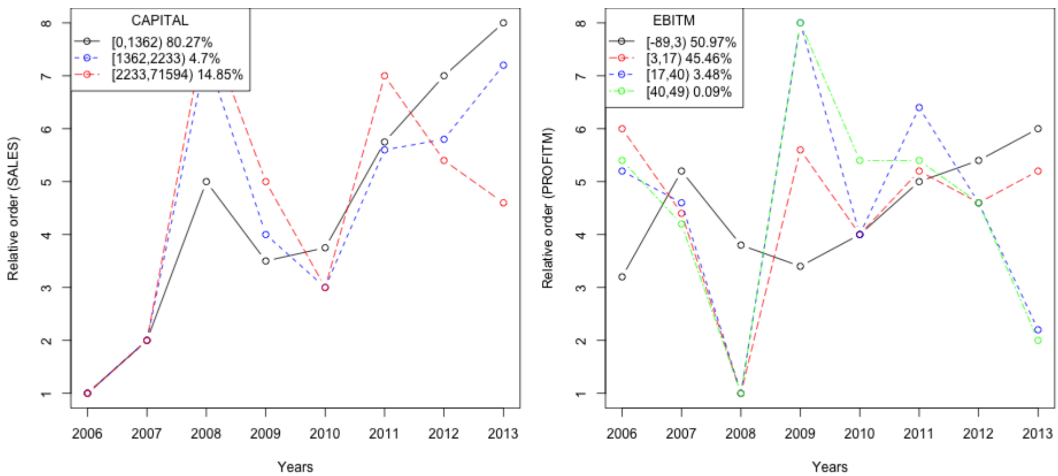


Figure 8: In the FOOD subsector: (a) CAPITAL related with SALES; (b) EBITM related with PROFITM.

that the crisis significantly affected firms in the food sector. However, if we observe

Fig 8, which shows CAPITAL vs SALES and EBITM vs PROFITM, we can see that firms with lower CAPITAL increased their sales after the crisis. Likewise, the firms with lower EBITM managed to improve their profit. Contrary, firms with higher EBIT ration decreased their relative order on the profit margin. Overall, we may conjecture that there were some kind of a Darwin type law, i.e. smaller capital and not so profitable firms that were able to survive the crises after all turn out to be more adapted and financially more efficient.

4 Concluding Remarks

We exploited the Multidirectional Efficiency Analysis to examine the effects of the financial crisis on the efficiency of the Portuguese firms dedicated to the manufacturing sector. Since 16 May 2011, Portugal has become the third Eurozone country, after Ireland and Greece, to receive international financial support to overcome financial difficulties, being a good candidate for studying such effects on firms. Our study involved seven manufacturing subsectors, divided into three time periods: pre-crisis (2006-2008), pre-troika (2009-2011) and troika (2012-2013). We adapted known mathematical methods, as much as possible, to every level of decision on the approach, e.g. use principal component analysis and dimensionality test to choose the "best" variables or use clustering analysis to find which are the adequate notion of "small" and "large" firm for the dataset. We also mention a reduction in the dimensionality of the usual MEA, in which we can study firm efficiency when there are good p-value fitting regressions. Furthermore, we presented two procedures from which is possible to visualize and make comparisons between efficiencies when it is possible to obtain regressions between variables with only one input variable and only one output variable. Basic statistics comparison between strategies of more efficient firms and less efficient firms were obtained by calculating the so-called NC-value.

Regarding the application of such methods, we addressed three empirical hypotheses:

H(1) *The performance of the portuguese firms in the manufacturing sector has been adversely affected by the financial crisis felt in Portugal in the troika years.* In addition to the MEA score, H(1) was tested by calculating the NC-values between more and less efficient firms. Although this hypothesis was confirmed, it was not totally and globally. In particular, the efficiency of firms decreased after the crisis for most of the subsectors except for TEXT and TRANSP. These subsectors showed the capacity to adapt to various changes brought by the crisis.

H(2) *Due to the financial crisis, the manufacturing sector acquired long-term debt deliberately.* This hypothesis was confirmed, see above. However, their use was not maximized, in general, as the inefficiency index for each variable show. In fact, LTDEBT was one of the inputs used most inefficiently in all subsectors during the three periods. Moreover, even with long-term debts acquired, many firms failed in improving their productivity, indicating that the resource was used to support unfavorable situations and not to improve their efficiency globally.

H(3) *The financial crisis has affected substantially the food subsector.* This hypothesis was rejected, see above. The food subsector was one of the few sectors that managed to maintain a stable performance during and after the crisis. Paradoxically although it was not pointed out, among the subsectors that improved their efficiency after the crisis, the food subsector did not suffer major decay in their economically-financial management. Furthermore, in this subsector there was an increase in the number of employees in the Troika period and, some firms, improved the solvency without resorting to huge debts.

While expected, that the negative effects of the European crisis would decrease over time, if the market attains some stability, in line with efforts to reduce the losses in all the manufacturing sector. Independent of the specificity of the economical situation in Portugal, the results of this study is a first step to have a broad picture that may improve the understanding about the capacity to react and overcome eventual economic difficulties.

It would be interesting to compare the results obtained in Portugal with manufacturing sectors of other European countries. This would involve working with very large data sets and therefore it requires a system with a highly computational demanding like the sDL package (see section 2). We allow the free use of the package rDATA/1.3, that we created for this study. Note that, in this study to apply the MEA method to 23K units, aggregated in different sets. Such produced more than 200 graphs, which are available at the web address http://tinyurl.com/l3fwrnm. Further conclusions may be extracted from those graphs.

In an extended point of view, our conclusions are still somehow limited in the sense that to precisely analyze the European crisis effects, other microeconomic, macroeconomic and sector specific variables are needed which will allow to better analyze the factors which explain the growth or the slowdown in the productivity, making it possible to further identify the strategy patterns followed by the firms to become more competitive in the globalized world.

Appendix The reader can check the section of the appendix which we refer in this work on *https://tinyurl.com/ycnvtcau.*

Acknowledgement. Work partially supported by Portuguese funds through the Center for Research and Development in Mathematics and Applications (CIDMA) and the Portuguese Foundation for Science and Technology (FCT), within project with reference UID/MAT/04106/2013; funded by Project 3599 – Promover a Produção Científica e Desenvolvimento Tecnológico e a Constituição de Redes Temáticas (3599-PPCDT) and FEDER funds through COMPETE 2020, Programa Operacional Competitividade e Internacionalização (POCI), and by national funds through FCT; Murillo is also supported by the FCT post-doc fellowship with reference SFRH/BPD/97085/2013 and Ramalho by the FCT grant UID/GES/00315/2013.

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