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Solomon Marcus, a mathematician for a lifetime

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Abstract: Our aim is twofold. First, we emphasize Solomon Marcus' close relation with the "Simion Stoilow" Institute of Mathematics of the Romanian Academy. Second, we present his interest in explaining the role of the mistakes in the evolution of the mathematical ideas.

Keywords: Solomon Marcus, mistake in mathematics.

MSC2010: 01A70, 01A60.

1 Introduction

Solomon Marcus deeply influenced the Romanian mathematical community, he constantly tried and succeeded to make mathematics visible as a main cultural activity. He was among the few mathematicians present in the media.

The goal of this paper is to underline several facets of the so complex personality of Solomon Marcus. Section 2 is devoted to the presentation of him as a professor and mentor, a special colleague at the Institute of Mathematics, a talented author of textbooks and popularization texts, a communicator of ideas in the public space.

In Section 3 we complete the contribution of Solomon Marcus from the volume "Şocul matematicii" (in Romanian) in discussing about the mathematical mistakes. We first summarize "the radiography of mistake" of S. Marcus: the mistakes of students in solving problems, about incorrect operations with correct results, mistakes that suggest and infer theorems and famous mistakes. Then we present three remarkable mistakes in mathematics: a first one pointed out in the monograph [C.S. Ogilvy and J.T. Anderson, *Excursions in Number Theory*, Oxford Univ. Press, 1966] and discussed by Solomon Marcus, the second one from Potential Theory, an error of C.F. Gauss, and the last one from the Population Dynamics, the birth of the Galton-Watson processes.

2 The art of becoming a nonagenarian [1]

Celebrating academician Solomon Marcus is an opportunity for Romanian mathematics to reveal the complexity of its cultural values. Solomon Marcus impersonates the ambassador of our mathematical community into intellectual fields which are apparently foreign to the rigor of exact sciences. It is no coincidence that he published, in a journal of applications of mathematics in architecture, an article entitled *The Art-Science Marriage.* We keep on trying to assemble fragments of an image of a personality, who deeply marks the Romanian culture.

Mathematician Solomon Marcus has always associated himself with the Institute of Mathematics of the Romanian Academy. Articles written six decades apart are signed with this affiliation, and the milestones of the history of the Institute are reflected in his academic activity. He is our colleague at the Institute and honours us by being involved in all our actions, from editing publications and organising conferences and congresses, to making decisions in the scientific council, but especially through his unmistakable interventions at public events.

He never means to be a comfortable interlocutor, exchanging ideas and dialogue are his preferences, he impresses us with his ability to take into account the initiatives and enthusiasm of others, mastering a skill of being a colleague that is difficult to match.

The internet revolution did not surprise him at all. Solomon Marcus learned, along with us, to communicate via email and, above all, adapted to the rapid circulation of information, to finding and storing it.

Solomon Marcus's mastery as a professor is easy to demonstrate. He influenced the education of the Romanian mathematical elite for several decades, and went far beyond the walls of the university, with chalk in hand, but also through textbooks and popularisation texts. For example, in such a book, Solomon Marcus manages to explain, with a minimum of formulas but rigorously, what analytic sets are and how they are constructed (the so-called Souslin scheme), related to the famous mistake of H. Lebesgue, regarding the projection of Borel sets.

The role of mistakes in the evolution of mathematical ideas became later a field of research for Solomon Marcus, complemented by major international recognition.

The 1980 edition of the famous *Mathematical Analysis Textbook* not only reflected the historical circumstances (being republished, without the authors' names), but was, for several decades, the Romanian standard in the field.

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[1] (a) This is the translation from Romanian of the article [L. Beznea, Arta de a ajunge nonagenar], published in *Academica* **293** (2015), no. 3, 53–54. (b) Speech delivered at the Tribute Session "Solomon Marcus - 90" (March 4th 2015, Romanian Academy Aula); translated in English by Anca Iuliana Bonciocat.

With remarkable intuition, Solomon Marcus added to the second volume a chapter on Nonstandard Analysis, which seemed, at the time, somewhat unusual. Three years later, however, the theory was to be presented in an invited conference of the International Congress of Mathematicians in Warsaw. The text remained unique in Romanian mathematical literature, and Nonstandard Analysis has, in the meantime, reached maturity. Today its limits are known, but its field of applicability is quite large.

The clearest proof of his talent as a teacher and mentor, however, is given by his disciples, scattered in all corners of the world: they know how to appreciate him at his true value.

Not far from the pedagogical talent is the impact he achieves through the ability of communicating ideas in the public space and carefully studying the reaction of the audience he addresses to. Behind it, however, lies a skillful work of informing himself, of filtering and eliminating noise, of then adding that epsilon, which brings the expected value.

In addition, he encourages us to follow his example, to try to become visible, not only in our community, but also in the press, on the radio and television.

Solomon Marcus showed creativity and sensibility as forms of art, starting with his first publications. Many of the Real Analysis articles published in the 1950s are little mathematical gems.

A common feature is the total lack of Markovianity in the evolution of the concepts he studied: the future (the open problems usually formulated at the end of the papers) depends on the past, not only through the present (i.e. the results obtained in the paper), but on the entire history of the problem. This is how quality mathematics is written.

A story:

In 1688, Abraham de Moivre emigrated to England, due to religious persecution. There he discovered *Principia Mathematica*, a text that he studied in detail, and had the chance to meet its author, Isaac Newton. A friendship was formed between the two, and de Moivre ended up being elected a member of the Royal Society, presided over by Newton himself. It is said that when he received questions about *Principia*, Newton would answer: "*Go and ask Mr. de Moivre, he knows these things better than I do*". I dare to imagine that to a Real Analysis question, Miron Nicolescu answered, at least once: "*Ask Mr. Solomon Marcus, he knows this field better than I do*".

Solomon Marcus continually offers us a unique demonstration of the art of being a mathematician for a lifetime.

3 Mistakes in mathematics[2,3]

Following the contribution of Solomon Marcus from the volume ”Şocul matematicii” (in Romanian) in analyzing the mathematical mistake, we outline three mistakes which influenced the evolution of the mathematical ideas. We first present several wrong arithmetical operations having correct results, mistakes which lead to mathematical theorems. Then we discuss a mistake of C.F. Gauss which influenced decisively the Potential Theory, a mathematical field located at the border between Mathematical Analysis, Partial Differential Equations, and Probability Theory. In addition, we emphasize Romanian contributions in connection with this error. Finally, we expose a mistake of H.W. Watson which put the British Empire in difficulty and was at the origin of the modeling of population dynamics, in particular when considering and studying the Galton–Watson stochastic process.

The radiography of mistake. In the volume “Şocul matematicii” (Editura Albatros, Bucureşti, 1987), Solomon Marcus dedicates a chapter to the topic of mistakes in mathematics. The chapter is entitled “The radiography of mistake” and has the following introduction: “*Social life is governed by operational rules, moral imperatives and legal norms. Their violation is usually sanctioned, but the nature and severity of the sanction are among the most varied, in relation to the nature and gravity of the violation. In science, especially in the so-called exact sciences, the distinction between correctness and mistake is most often rigorous.*”

Marcus then talks about the mistakes of students or candidates in solving problems in competitions and exams, about establishing the grading scales, about incorrect operations with correct results, mistakes that suggest and infer theorems and famous mistakes: “*the mistake of Augustin Louis Cauchy who believed that any convergent series of continuous functions on a segment (a, b) has as its limit a continuous function on (a, b). The road to the crystallization of the concept of convergence in any form went through Cauchy's mistake. Tomas Stieltjes, the author of the integral bearing his name, believed that he had proved the famous Riemann hypothesis.*”

Solomon Marcus presents in detail the famous mistake of Henri Lebesgue (from a work published in 1905), the creator of the famous integral. The mistake had been

[2](a) This is the translation from Romanian of the article [L. Beznea, Greşeli în matematică], published in *Didactica Matematică* **14** (2024), Nr. 1, 1–6. (b) Presentation held at the 25th Annual Conference of the Romanian Mathematical Society, organized by the Botoşani Branch of the Romanian Mathematical Society at the ”Grigore Ghica” National College in Dorohoi, May 12-14, 2023, dedicated to the 150th anniversary of the birth of Dimitrie Pompeiu.

[3]In the text, several quotes from the volume ”Şocul matematicii” [The Shock of Mathematics] by Solomon Marcus are placed between quotation marks; translated in English by Anca Iuliana Bonciocat.

detected by Mikhail Suslin in 1917, a student of Nikolai Luzin. Suslin died very young, in 1919, of typhus, following the Russian Civil War. Lebesgue believed that the projection onto the x-axis of any Borel set from the plane is still a Borel set on the line. Suslin showed that such a simple operation as the projection, applied to a Borel set, can lead to a non-Borel set. Through Lebesgue's mistake, the first example of an *analytic set* was given, thus the *Analytic Set Theory* emerged, a field further developed by Luzin. Luzin actually proved that Lebesgue's operation on a Borel set is correct if instead of a projection we have an injective function, a mathematical result which is known today as the famous *Luzin Theorem*. The insistence here on this mistake is justified by the well-known contributions of the Romanian School of Potential Theory in the field of Analytic Set Theory and, further, of the Theory of Choquet capacities, contributions initiated by our professors Nicu Boboc, Gheorghe Bucur and Aurel Cornea, but also by Corneliu Constantinescu.

Marcus notices also a Romanian contribution on mathematical error, a 1971 paper by Alexandru Froda (1894–1973), professor at the University of Bucharest and researcher at the Institute of Mathematics of the Romanian Academy.

"*How often do mathematicians make mistakes?*" Joseph L. Doob (1910–2004), founder of Martingale Theory and Probabilistic Potential Theory, stated that: "*If you open a mathematical work at random, it is very likely that there will be at least one mistake there. Do mathematicians make more mistakes than other specialists? Is mathematics more error-prone than physics, chemistry, biology, or geology? Probably not. ... But in mathematics, more than in other fields, the boundary between correctness and mistake is, most of the time, precise, indisputable.*"

We now present three remarkable mistakes in mathematics. The first was pointed out by C.S. Ogilvy and J.T. Anderson in their monograph *Excursions in Number Theory*, Oxford Univ. Press, 1966, and was also cited by the already mentioned work of Solomon Marcus. The second and third, however, are known only to experts in Potential Theory and, respectively, Population Dynamics.

1. Incorrect operations with correct results, errors that suggest theorems. "What would you say if considering the fraction 16/64 a student would want to "simplify" 6 in the numerator with 6 in the denominator? An enormity leading to a failing grade, right? And yet the result of this ridiculous simplification operation is correct because

$$16/64 = 1/4.$$

Wrong thinking does not always lead to a wrong result, at least not immediately. Of course, if the person in question would persist in mistake by applying, for example, the same treatment to the fraction 12/24 and "simplify" 2 in the numerator with 2 in the denominator, then he would get the wrong result 1/4. Wrong behavior does not always lead to the same outcome. A wrong outcome is symptomatic of a wrong treatment, it warns us that we are not reasoning correctly, just in the same way that pain is symptomatic of a disease state. Painless diseases are dangerous precisely because they lack the signal that should draw our attention to the need to undergo a medical examination and appropriate treatment (which may also include a change in behavior). Equally dangerous is the vicious thinking that does not affect the outcome; it can deceive us with the illusion of correctness."

Let us return to the simplification of 6 in the fraction 16/64 to obtain 1/4. Note that a similar incorrect simplification, applied to the fraction 26/65, leads to the correct result 2/5,

$$26/65 = 2/5.$$

Using decimal notation, we have:

$$(10 \cdot 1 + 6)/(10 \cdot 6 + 4) = 1/4$$

and

$$(10 \cdot 2 + 6)/(10 \cdot 6 + 5) = 2/5.$$

Let us try now to generalize this example: *For what positive integer values of a, b and c does the equality hold*

$$(10 \cdot a + b)/(10 \cdot b + c) = a/c$$

or equivalently

$$9ac = b(10a - c)?$$

A trivial solution is $a = b = c$. The example above does not belong here, but refers to the case where a is different from c, and $b = 6$.

We obtain the equation

$$(10 \cdot a + 6)/(10 \cdot 6 + c) = a/c$$

having the two solutions mentioned above:

$$a = 1, c = 4 \text{ and } a = 2, c = 5.$$

In the monograph by C.S. Ogilvy and J.T. Anderson, on page 86, this example is discussed and it is demonstrated that these are the only solutions.

2. An error of Gauss. In a work on the history of Potential Theory [La théorie moderne du potentiel, *Annales de l'Institut Fourier*, 4 (1952)], Marcel Brelot discusses an error by Carl Friedrich Gauss (1777–1855). It is unnecessary to justify why Gauss is one of the greatest personalities in history, a genius of mankind. I had the opportunity to point out this error to Solomon Marcus himself. Marcus simply did not believe it was possible, Gauss being recognized as one of the few mathematicians who would never make a mistake. Following Gauss's brilliant ideas, at the boundary between mathematics and physics (in particular, electrostatics), Brelot founded in the middle of the last century what we call today the *Axiomatic Potential Theory.* Brelot does not hesitate to emphasize the fundamental role played by Gauss but mentions:

Mais si l'on peut dire que tout est dans l'oeuvre de Gauss, il apparut bientôt que la ***rigueur*** *était insuffisante. D'abord Gauss admettait que le minimum de l'intégrale est réalisé pour une distribution de masses; et comme il ne considérait que les distributions pourvues de densité, cela ne pouvait être vrai sans restrictions sur la surface. De plus, nous savons aussi maintenant que sans de telles restrictions, le problème d'équilibre n'est pas exactement résoluble. Pour surmonter ces difficultés et parvenir à des résultats généraux il fallut attendre près de cent ans et arriver à la thèse de Frostmann (1935).*[4]

This intervention of Brelot is a jewel of language, a delicate explanation of an inaccuracy of a titan of science, in a profound and precise mathematical text. Here we find the germs of the Balayage Theory, a central chapter of Potential Theory. In particular, it features measures "without a density", the Stieltjes integral can produce such measures, but here the example has a physical phenomenon behind it, coming from electrostatics.

We emphasize M. Brelot's observation that only after eight decades Gauss's mistake was completely understood. A few years later Marcel Brelot completed the notes on the history of Potential Theory and highlighted the essential contribution to the theory of the Boboc–Constantinescu–Cornea "triplet". It is the beginning of the international visibility of the Romanian School of Potential Theory.

3. The mistake that set an empire on fire or the origin of population dynamics modeling. The study of population evolution over time was born in the 19th century, out of interest in the possible disappearance of noble names in England. Sir Francis Galton (1822–1911, initiator of eugenics, half-cousin of Charles Darwin) studied the disappearance of noble names by using statistical methods. In 1873, Francis Galton proposed in the magazine *The Educational Times*, **25** (1873), on page 300, under the heading ,,Questions for solutions", the following problem (Problem 4001):

[4]But if we could say that everything was contained in Gauss's work, it soon became clear that the **rigor** was insufficient. First, Gauss admitted that the minimum of the integral is achieved for a mass distribution; and since he only considered distributions provided with a density, this could not be true without restrictions on the surface. Moreover, we also know now that without such restrictions, the equilibrium problem is not exactly solvable. To overcome these difficulties and obtain general results, it was necessary to wait almost a hundred years, until Frostmann's thesis (1935).

A large nation of whom we will only concern ourselves with the adult males, N in number, and who each bear separate surnames, colonize a district. Their law of population is such that, in each generation,

a_0 *per cent of the adult males have no male children who reach adult life;*

a_1 *have one such male child;*

a_2 *have two; and so on, up to*

a_5 *who have five.*

Find: (1) what proportion of the surnames will have become extinct after r generations; and (2) how many instances there will be of the same surname being held by m persons.

The problem proposed by F. Galton received only one solution, given by Henry William Watson (1827–1903), a pastor who later became a member of the Royal Society. In fact, the solution was wrong, Watson not being aware of the complexity of the problem. More precisely, let us first note that

$$a_0 + a_1 + a_2 + a_3 + a_4 + a_5 = 1.$$

If we denote by x the probability of extinction of male descendants, then x satisfies the equation

$$x = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5.$$

Watson observed that $x = 1$ is a solution to this equation and wrongly deduced that we always have, with certainty, the extinction of male descendants and, therefore, the disappearance of family names.

F. Galton and H. Watson then wrote together the work "On the probability of the extinction of families", published in 1874 in the *Journal of the Royal Anthropological Institute.*

It is the beginning of a new theory, of *branching processes.* The process that describes the phenomenon in the problem proposed by F. Galton has been named *Galton–Watson* and is a classic one in the probabilistic modeling of the evolution of populations over time. It can be shown today, using J.L. Doob's Martingale Convergence Theorem, that, in fact, in certain situations Watson was right; we can have the extinction of descendants, especially in the "subcritical" case.

The relevance of Galton–Watson-type processes is remarkable. Over the past five decades, these processes have become relevant mathematical models in biology, in particular in genetics. It is found that the evolution over time of a DNA macromolecule, the appearance of "replicas", is similar to the phenomenon of the disappearance of surnames, a problem studied in the monograph [D.E. Axelrod, M. Kimmel, *Processes in Biology*, Springer, 2015].

Richard Durrett, a living classic of Probability Theory, published the paper [*Branching Process Models of Cancer.* Mathematical Biosciences Institute Lecture

Series, vol. 1.1, Springer, 2015]. This study is of fundamental importance and uses Galton–Watson process as a basic tool (see section 3.1 of the paper).

Watson's mistake, the Galton–Watson process, and general branching processes have regained interest after the pandemic we have just experienced. An overview of the use of branching processes in the study of epidemics can be found in the article [T. Britton, Stochastic epidemic models: a survey, *Mathematical Biosciences*, **225** (2010)]. Note that the use of Markov branching processes is compatible with the rules of natural epidemic evolution: the **future** (the population that will become infected) depends on the **past** (the infected population) only through the **present** (the currently contagious population). Since it is accepted that an individual can infect a number of contacts with varying probabilities, branching processes become appropriate models for the early stages of epidemics (when interactions have not yet occurred) or when the epidemic is controlled by mass vaccination, at key moments determined by the epidemic model; see the paper [C.P. Farrington, M.N. Kanaan, N.J. Gay, Branching process models for surveillance of infectious diseases controlled by mass vaccination, *Biostatistics*, **4** (2003)].

To sum up, mistakes cannot be separated from the evolution of mathematical ideas, their clarification and examination is part of the history of mathematics, "valuable ideas must pay the tribute of insufficient rigor".

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