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I S A A C J. S C H O E N B E R G

Isaac J. Schoenberg was born on April 21 , 1903 , in the city of Galatz (Galaţi) , Romania . His father , Jacob Schoenberg (1864 - 1930) was an Austro-Hungarian subject at that time , and together with his family adopted the Romanian citizenship after the WWI . He was an accountant by training , and held several positions in Romania , including that of Vice President of the "Banca Moldova" , and that of Consul of the Austrian Empire in Jassy (Iaşi) , where the family moved in 1910 . The mother of I.J.Schoenberg , Rachel Schoenberg (Segal) , was known as a fine Romanian poetess , who also wrote and spoke French fluently . Both parents were devoted Zionists , with the mother a public speaker at the Zionosts meetings , and the father active in establishing agricultural stations for the training of the future citizens of the State of Israel .

I.J.Schoenberg attended school in Jassy , and during his years in High School he has been particularly attracted by subjects like Physics and Mathematics . One of his Mathematics teachers , Titu Patriciu , has exerted a strong influence on the young Schoenberg , who decided to dedicate his life to the study of Mathematics . At the age of 16 , in 1919 , I.J.Schoenberg passes the Baccalaureate examination , making himself eligible for college .

In the Fall of 1919 , I. J. Schoenberg became a student with the University of Jassy , the first Romanian university which has been founded in 1835 under the name "Academia Mihaileana" . Until 1922 , when he obtained the Bachelor of Science degree in Mathematics , I. J. Schoenberg has taken all the courses required by the university regulations at that time : Projective and Descriptive Geometry (Victor Costin) ; Analytic Geometry (Alexandru Myller) ; Algebra with Galois Theory (Simion Stoilow) ; Mathematical Analysis (Simion Sanielevici) ; Theory of Analytic Functions (Vera Myller-Lebedeff) ; Mechanics (Simion Saniele -

vici) ; Astronomy (Constantin Popovici) . Outside the "Licenta" program , I.J.Schoenberg has taken courses on Differential Geometry (A.Myller), and Differential Equations , with special regard to Fuchsian Theory (S.Sanielevici) . The Myllers were students in Goettingen at the beginning of the Century , and they obtained their Doctoral degrees under Felix Klein (A. Myller) and David Hilbert (Vera Myller-Lebedeff) . Simion Sanielevici has obtained his Ph. D. degree in Paris , with Emile Picard as supervisor . Constantin Popovici has also obtained his Ph. D. degree in France , and despite his association with Astronomy , he was an active researcher in the theory of functional equations . The Myllers are considered as the founders of the Mathematical Seminar at the University of Jassy , and it is definitely true that mathematical research was developed at that university only after 1910 , the year in which Alexandru Myller has been appointed a Professor of Analytic Geometry .

The next three years , from 1922 until 1925 , I.J.Schoenberg was a student in Germany . After one semester spent at the University of Goettingen , he moved to the University of Berlin where he remains for three consecutive semesters . The last two semesters in Germany have been spent at the University of Goettingen . Among the courses taken in Germany there have been those given by Edmund Landau (Entire Functions , Trigonometric Series , Analytic Number Theory) , Issai Schur (Algebra , Number Theory , Analytic Number Theory) , and A.Ostrowski (Superconvergence of Power Series) . I.J.Schoenberg has also attended a Problem Seminar offered by A. Ostrowski and K.Granjot . During the period spent in Germany , I.J. Schoenberg has accumulated valuable knowledge and experience in research . The main idea of his Ph. D. thesis has been suggested by Schur's course in Analytic Number Theory , and it is related to H.Weyl's theory of uniform distribution of numbers mod. 1 . Schur gave some new asymptotic properties of Euler's function $\phi(n)$, which suggested to Schoenberg the more general theory of non-uniform distribution of numbers mod. 1 . This new theory has been initiated in his Ph. D. thesis . One of the main results obtained in this direction is concerned with the asymptotic distribution function $F(x)$ of the particular sequence $x_n = \phi(n)/n$, $n = 1 , 2 , 3 , \ldots$ He has shown that $F(x)$ is different from x and is continuous and increasing in $[0,1]$. In 1939 , Paul Erdös has shown that $F(x)$ is a singular

function , i.e. $F'(x) = 0$ almost everywhere . This is perhaps the most natural example of a singular function in the sense of Lebesgue .

Due to the fact that the Prussian Ministry of Education did not recognize the High School Diploma Schoenberg has obtained in Romania , it was not possible to obtain a Ph. D. degree while in Germanuy. In the Summer of 1925 I.J.Schoenberg returned to Romania . and has been appointed an Assistant to Professor Victor Costin . In June 1926 , I.J.Schoenberg defended his Ph. D. thesis at the University of Jassy , under Professor Simion Sanielevici . After publication , I.J.Schoenberg sent the thesis to I.Schur , who in 1928 has sent a letter to Schoenberg , stating : "Herr Doktor , in this matter you are ahead of everybody, you must utilize this advantage" .During the period 1925 - 1930 , I.J.Schoenberg was associated with the University of Jassy , excepting the year 1926 - 1927 when he served in the military , attending the Field Artillery School in Timişoara , and then spending half a year in the city of Chişinau (Kishinev , nowadays in Soviet Moldavia) with a Regiment . He has been another semester on leave , during the Spring of 1928 , when he taught an Algebra course at the recently created Hebrew University of Jerusalem . There , at the Einstein Institute , he met his former Professor from Goettingen , Edmund Landau , whose daughter Charlotte became Schoenberg's spouse in 1930 .

For the academic year 1930 - 1931 , I.J.Schoenberg was granted a Rockefeller Fellowship , and he has chosen to spend the year at the University of Chicago in view of studying the Calculus of Variations under the direction of Gilbert Bliss . The following year has been also spent at the University of Chicago with an Assistanship granted by Professor Bliss . The Schoenberg family , to which a dauther (Elizabeth) has been added , moved to Cambridge MA for the academic year 1932 - 1933 , where courses taken with David Widder at Harvard , and with Dirk Struik and Jesse Douglas at M.I.T. completed the already extensive training of I.J.S. The next three semesters have been spent in Princeton , where I.J.Schoenberg has become a Fellow of the newly created Institute for Advanced Study . He has also helped with the publication of the Annals of Mathematics . During his time in Princeton , I. J.Schoenberg became interested in Distance Geometry (initiated by Karl Menger in Vienna). Stimulated by a note of Maurice Fréchet in Annals of Mathe-

matics , I.J.Schoenberg starts a research program that kept him busy for several years , during which period he cooperated and produced joint results with such illustrious mathematicians like J.von Neumann . This series of papers is dedicated mostly to the problem of isometric imbedding of metric spaces in the Hilbert space , and to its connections with the Analysis . The dominant ideas where those of Positive Definite Function and Isometric Imbedding . Distinguished mathematicians like G.Choquet , N.I.Akhiezer , A.M.Yaglom , Chr. Berg , J.Christensen and P.Ressel have mentioned and discussed the results obtained in this field by I.J.Schoenberg , during the Princeton period .

In January 1935 , I.J.Schoenberg obtained his first teaching job in the United States . Upon recommendation given by Oswald Veblen , he has been assigned a position of Acting Assistant Professor at Swarthmore College in Swarthmore PA , where he remained for three semesters . This time, upon recommendation of Marston Morse , he obtained a position at Colby College in Waterville ME , where he moves with his family in 1936 . They remained there for the next five years , a period which is characterized by I.J.Schoenberg as "five enjoyable years" . Besides teaching duties consisting of giving courses in Mechanics and other disciplines , I.J.Schoenberg continued his work in Distance Geometry .

In 1941 , John Robert Kline offered I.J.Schoenberg a position at the University of Pennsylvania . Excepting some interruptions during the WWII and the sabbaticals , I.J.Schoenberg remained with this university until 1965 . He served as Chairman of the Department of Mathematics at the University of Pennsylvania from 1960 until 1963 . During his chairmanship , I.J.Schoenberg organized a Symposium on Number Theory , in honor of Hans Rademacher . While at University of Pennsylavania , I.J.Schoenberg taught courses on Complex Analysis , Real Variables , Functional Analysis , Laplace Transform Theory , and Approximation Theory . Jointly with H.Rademacher , a Problem Seminar has been conducted , and at least 10 Ph. D. students have worked under I.J.Schoenberg's supervision .

During the period 1943 - 1946 , I.J.Schoenberg has been called to join the war effort . The first electronic computer , the ENIAC ,was under development at the Moore School of the University of Pennsylavnia . Leo Zippin was in charge with the project (he was a Corporal !) . The project had to be moved at the Ballistic Research Laboratory (the BRL) in Aber -

deen MD . I.J.Schoenberg had to join the staff at the BRL , and he continued to work there until 1946 . It is this work place which stimulated I.J.Schoenberg in achieving the most celebrated results in his career. But let himself speak about the circumstances that led him to introduce in research what has been subsequently called Spline Approximation , and what proved to be one of the most efficient concepts of Approximation Theory in Computer Era : "The morning in August 1943 of my reporting for duty , Major A.A.Bennett of Brown University , then Chief of the Computing Branch of the BRL , told me what my particular problem was to be : Trajectories of projectiles were until then computed at desk calculators by hand.Into these computations entered tables of the drag-functions of air resistance , about 24 of them , which were obtained empirically by firing of various types of projectiles . As the step of integration used in these computations was rather large and the methods of numerical integration fairly complicated , it did not much matter that the 4-place drag-function tables were rather rough . In performing these computations on the ENIAC , which was very fast , a much simpler integration method of very small step could be used .In these methods , the accumulation of the round-off errors was unacceptable due to the rough drag-function tables . They needed to be smoothed by being approximated by analytic functions . To do this was my problem . I solved this problem by what later I called Cardinal Spline Interpolation and Cardinal Spline Smoothing . To insure the smoothness , to 8 decimal places , of the second derivative of the approximant of the drag-function , I used the heat-flow equation on the real axis to produce regular approximations on the entire real axis . Only the fine resources of the BRL made this work possible". As pointed out by I.J.Schoenberg , "Equidistant splines were used before me , e.g., Runge (quadratic splines , 1901) , and Quade and Collatz (arbitrary degree , 1938) for the approximation of the Fourier coefficients of periodic functions , but not for the approximation of functions".

In January 1946 , I.J.Schoenberg returned to the University of Pennsylvania . During the academic year 1956 - 1957 he went on a sabbatical at Stanford University , to work with Polya . Among the results of the joint work at Stanford , a conjecture has been advanced . It has

remained a conjecture for the next 15 years , known as Polya-Schoenberg conjecture . In 1973 , S.Ruscheweyeh and T.Sheil-Small (Comment.Math. Helv. , vol. 48) have proven the conjecture . Another fine mathematician with whom I.J.Schoenberg cooperated during his years at University of Pennsylvania was A.S.Besicovitch , who retired from Cambridge University and then became a member of the department at Penn . During the year 1963 - 1964 , I.J.Schoenberg takes another sabbatical at the Institute for Advanced Studies in Princeton . Here , he concentrate his efforts on the Theory of Spline Functions , and its connections with other branches of Analysis . This intensive activity has been continued during one more year , at the Mathematics Research Center of the University of Wisconsin at Madison . As I.J.Schoenberg points out in his "Brief account of my life and work" , "I had the spline functions for myself from 1946 to a - bout 1960 , when they were rediscovered by several people ". Making clear the priority in this field , was his primary concern for the period spent at Princeton and Madison . Also during his tenure at Penn , I.J. Schoen - berg obtained a leave for the academic year 1951 - 1952 , and spent this year at U.C.L.A. , at the Institute of Numerical Analysis of the National Bureau of Standards . At that Institute , I.J.Schoenberg actively coope- rated with Th. Motzkin . An anticipation of multi-dimensional splines has appeared during this work , but this concept will be reconsidered by W. Dahmen and C.A.Micchelli later on (J.d'Analyse Math. , vol. 39).

Meantime , changes occured in the family life . In 1949 , I.J.Schoen- berg's spouse Dolli (Landau) died of acute leukemia . Seventeen months later , in December 1950 , I.J.Schoenberg is remarried to Dolly van der Hoop of Amsterdam , The Netherlands . Their son Michael Jan was born in Santa Monica CA in September 1951 .

In june 1965 , I.J.Schoenberg resigned his position at Penn , and ac- cepted a similar position at the University of Wisconsin , joining both the Mathematics Department and the MRC . He remained on active duties with this school for the next 8 years , retiring in June 1973 . During those years , the main topic of his research was the Cardinal Spline Interpolation . Se- veral younger researchers , such as P.R.Lipow , A.Sharma and Carl de Boor , have contributed to the progress of this domain of investigation . The main results pertaining to this period have been summarized in the monograph

edited by I.J.Schoenberg "Approximations with Special Emphasis on Spline Functions" , published by Academic Press in 1969 . In the sixties , Spline Theory became one of the most active topics in the Theory of Approximation. In 1967 , the first book dedicated to splines is published by J.H.Ahlberg, E.N.Nilson and J.L.Walsh at Academic Press : "The Theory of Splines and Their Applications" . This book has been translated into Russian by Yu. N.Subbotin . This was followed by the volume in the MRC Symposia series T.N.E.Greville (ed) , "The Theory and Applications of Spline Functions" , Academic Press , 1969 . More recently , more specialized books have been published on this subject : Carl de Boor , "A Practical Guide to Splines", Springer-Verlag , 1978 , and L.L.Schumaker , "Spline Functions:Basic Theory", John Wiley & Sons , 1981 . During his tenure at Madison , I.J. Schoenberg supervised the following Ph. D. students : D.R.Ferguson , Alfrd Cavaretta , F.R.Loscalzo , Martin Marsden , Franklin Richards , Sherwood Silliman and I. Yuan Haung .

I.J.Schoenberg visited many countries and institutions of Higher Learning while at University of Wisconsin , and after retirement :In 1966 he attended the International Congress of Mathematicians in Moscow , where he met for the first time M.G.Krein , after many years of correspondence . In 1967 he spent two months in Europe , visiting Romania for the first time since 1930 . He lectured in Bucharest , Cluj and Jassy , being the guest of the Romanian Academy . I was in Jassy at the time , and I do remember the excitement produced by his coming back to Alma Mater after 37 years of absence . His lecture at Jassy was given in Romanian . We have been delightfully surprised to find out that it was perfect. However,one of our colleagues noted that in regard to the space L^2 , I.J.Schoenberg pronounced L - two , instead of L - doi as we were saying in Romanian . It was also very difficult for many of us to understand how did I.J.Schoenberg hold positions with several universities in the USA , for almost fourdecades , while his Ph. D. Diploma was still at the University of Jassy . On that occasion , I.J.Schoenberg retrieved his Diploma . In 1969 he visited Israel for three months , lecturing in Haifa , Jerusalem , Tel-Aviv , and at the Weizmann Institute in Rehovot . In 1977 he is in Scotland , for one month at the University of Dundee . In 1977 he was in Rome , with the Istituto "Mauro Picone" , and has given lectures in Florence and Pisa . His concern in Rome was with Cardinal Spline Smoothing .

Besides the above mentioned institutions , I.J.Schoenberg has visited for at least one semester the following places : The University of California at San Diego ; The University of Pittsburgh ; The United States Military Academy at West Point NY (June 1977 - June 1978) . The weekly seminar he organized and directed at West Point let him to the publication of the book "Mathematical Time Exposures" , Math. Assoc. of America , 1983 . The author has dedicated this book to the memory of his Professors at the University of Jassy .

I.J.Schoenberg has brought significant contributions to different other topics . This is a very succint presentation of his life and work , which implies neglecting some of those topics . However , we would like to mention a few of those falling in this category . First of all , is the solution of a problem formulated by Polya in 1915 , concerning entire functions . An integral representation formula for the inverse of certain functions is obtained , involving totally positive functions . The so-called Polya frequency functions are introduced and investigated. Another significant contribution is related to the Landau-Kolmogorov inequality , and its connections with extremum problems . Finally , pertaining to the Geometry of Numbers is a series of contributions , of a recent date , concerning extremal problems for billiard ball motions in cubes .

This year (1988) , with Carl de Boor as editor , Birkhauser-Verlag has published two volumes of Selected Papers , containing the most important contributions I.J.Schoenberg has made during his remarkable career , spanning more than six decades , thus covering a sizable fraction of what we call Modern Mathematics .

Note . The above presentation has been compiled by C.Corduneanu , using I.J.Schoenberg's "A brief account of my life and work" .