LIBERTAS MATHEMATICA , VOL. 8 (1988)

# EFFECT OF TURBULENCE ON DARRIEUS ROTOR

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## Abstract

The influence of turbulent wind on Darrieus wind turbines has recently emerged as significant consideration for wind turbine design and optimization. The purpose of the present paper is to study the influence of atmospheric turbulence on Darrieus rotor performance by using stochastic wind model into the Double-Multiple Streamtube aerodynamic model (DMS).

Theoretical predictions of the aerodynamic loads were performed for Sandia 17-m wind turbine and compared with the available experimental data and CARDAAV results. This Comparison shows an important improvement of the loads predicted by using stochastic wind. These loads agree well with experimental data.

## Nomenclature

$a$, $a'$ = interference factors
$c$ = blade chord, m
$C_D$, $C_L$ = section drag and lift coefficients
$C_N$, $C_T$ = blade airfoil normal and tangential force coefficients
$f_{up}$ = upwind functions, Eq. (18)
$F_N$, $F_T$ = normal and tangential forces
$h$ = height above ground, m
$H$ = half height of the rotor, m
$n$ = frequency, Hz
$N$ = number of blades
$N_p$ = number of points in the time-series
$R$ = rotor radius at the equator, m
$R_{eb}$ = blade Reynolds number
$T$ = torque, Nm
TSR = tip speed ratio
$u$, $v$ = fluctuation velocities, m/s
$V$ = local velocity, m/s
$V''$ = wake velocity, m/s
$V_D$, $V_U$ = downwind and upwind induced velocities
$V_f$ = normalized fluctuation velocity
$W$ = relative velocity, m/s
$x$ = streamwise velocity location
$x_{s0}$ = streamwise position at which the turbulence is generated
$Z_i$ = local turbine height, m
$\alpha$ = local angle of attack, deg.
$\alpha_W$ = atmospheric wind shear exponent
$\Phi$ = nondimensional power spectral density

$\phi(n)$ = power spectral density
$\delta$ = angle between the blade normal and the equatorial plane
$\zeta$ = nondimensional cartesian coordinate = z/H
$\eta$ = nondimensional cartesian coordinate = r/R
$\eta^+$ = reduced frequency
$\Delta\eta^+$ = dimensionless frequency band
$\nu_\infty$ = kinematic viscosity, $m^2/s$
$\sigma^+$ = dimensionless turbulence intensity
$\tau^+$ = dimensionless time
$\vartheta$ = azimuthal angle, deg.
$\rho$ = freestream density, $kg/m^3$
$\omega$ = turbine rotational speed, $s^{-1}$

subscripts

$\infty$ = freestream conditions
$\infty_i$ = local conditions in the vertical directions

## 1. Introduction

During the design of wind turbine the early studies were mainly concerned with aerodynamic performance and optimization of wind turbines to achieve economic target. As these problems are overcome, actually the main area in which better analytical tools are required deal with stochastic wind and aerodynamic effects modeling and better fatigue life predictions. Wind turbine blades have been known to fail due to the structural fatigue after sometimes as little as 2-3 years despite having been designed for a fatigue life and it was identified that atmospheric turbulence is one of the major source of the aerodynamic loads [1]. The steady-state aerodynamic code on which these designs are based assume a constant ambient wind which does not vary with time. As a result, the predicted loads on the blade are identical for each rotor revolution and we have no informations about the effect of turbulence on the rotor. The investigation herein provides one tool to estimate these effects by introducing a stochastic wind model into the DMS aerodynamic model.

## 2. Wind simulation

The velocity field of the wind is assumed to be a linear superposition of a steady or mean component and a stochastic fluctuation. The one dimensional variations of the turbulent wind are produced by creating wind time series at a fixed point upwind the rotor and assuming that the wind speed is constant in a plane perpendicular to the mean wind direction. The turbulent wind speed at downstream of the fixed point is then obtained by calculating a time delay in the time series. The decrease in the streamwise velocity as the flow passes through the rotor is taken into account by assuming a linear variation in the streamwise direction [2]. The power spectral density (PSD) used is given by [3]:

$$\phi_w(n) = \left[\frac{\sigma^2}{n}\right] \frac{0.164\,(\eta^+/\eta_0)}{1 + 0.164\,(\eta^+/\eta_0)^{5/3}} \tag{1}$$

In order to satisfy the relationship between the turbulence intensity and PSD, the nondimensional power spectral density Φ, is given by:

$$\Phi = \frac{\phi_w(n)V_{\infty i}}{h\,\sigma^2} = \frac{0.171/\eta_0}{1 + 0.164(\eta^+/\eta_0)^{5/3}} \tag{2}$$

The fluctuation velocities u and v due to the turbulent wind are represented by a Fourier time-series:

$$V_f^+ = \sigma^+ \sum_{j=1}^{N_p/2} \left[ A_j^+ \sin(2\pi\eta_j^+\tau^+) + B_j^+\cos(2\pi\eta_j^+\,\tau^+) \right] \tag{3}$$

where

$$V_f^+ = V_f / V_{\infty i} \tag{4}$$

$$V_f = u,\ v \tag{5}$$

$$A_j^+ = (2\ \Phi_j\ \Delta\eta^+)^{1/2}\ \sin\phi_j \tag{6}$$

$$B_j^+ = (2\ \Phi_j\ \Delta\eta^+)^{1/2}\ \cos\phi_j \tag{7}$$

$$\tau^+ = \tau V_{\infty i}/h \tag{8}$$

**With the above relations, values of the fluctuation velocities are performed by using the Fast Fourier Transform. For the moving nodes the time delay $\Delta\tau_D^+$, at any position x, is given by:**

$$\Delta\tau_D^+ = \frac{V_{\infty i}}{h}\int_{x_{s0}}^{x} \frac{dx}{V(x)} = \frac{r}{h}\left[\frac{1}{c_2}\ln(c_1 + c_2\frac{x}{r}) - \frac{x_{s0}}{r} - 1\right] \tag{9}$$

**where the coefficients $c_1$ and $c_2$ are a function of the wake velocity $V''$ and are related as:**

$$c_1 + c_2 = \frac{V''}{V_{\infty i}} \tag{10}$$

$$c_1 - c_2 = 1 \tag{11}$$

## 3. Aerodynamic model

A large number of aerodynamic prediction models currently exist for studying Darrieus wind turbine and a complete state of the art, including the appropriate references, is given by Refs. [4], [5]. Generally speaking, the fundamental objective of all aerodynamic models is to evaluate the induced velocity field of the turbine since knowledge of this velocity field allows all the forces on the blade and the power generated by the turbine to be determined. Since time-domain analysis require long record lengths, to reduce statistical scatter the Double-Multiple Streamtube model (DMS) was chosen. In this model the upwind and downwind components of the induced velocities at each level of the rotor are calculated by using the principle of the two actuator disks in tandem as described by Ref. [6]

The freestream velocity profile is calculated from the following equation:

(12) $$V_{\infty i}/V_{\infty} = (Z_i/Z_{EQ})^{\alpha w}$$

The upwind velocity component is less than the local ambient wind velocity and in the middle plane between the upwind and downwind zones there is an equilibrium-induced velocity. Velocities at the upwind region, middle plane and downwind region are given by:

(13) $$V_u = a\, V_{\infty i}$$

(14) $$V_e = (2a-1)V_{\infty i}$$

(15) $$V_D = a'V_e = a'(2a-1)V_{\infty i}$$

where "a" and "a'" represent the upwind and downwind interference factors. Using the blade element theory and the momentum equation for each streamtube and equating the vertical variation of the induced-drag coefficient of the rotor, its found that:

(16) $$f_{up}(V/V_{\infty})^2 = \pi\eta(V/V_{\infty})\left[(V_{\infty i}/V_{\infty})-(V/V_{\infty})\right]$$

or, in terms of the interference factor,

(17) $$f_{up}a = \pi\eta(1-a)$$

where $\eta = r/R$, and $f_{up}$ is the upwind function which characterizes the upstream half-cycle of the rotor on the blade element rotating in this zone. Based on the forgoing consideration, the upwind function is given by:

(18) $$f_{up} = \frac{Nc}{8\pi R}\int_{-\pi/2}^{\pi/2}\left[C_N\frac{\cos\theta}{|\cos\theta|} + C_T\frac{\sin\theta}{|\cos\theta|\cos\delta}\right](W/V)^2 d\theta$$

where

(19) $$C_N = C_L\cos\alpha + C_D\sin\alpha$$

(20) $$C_T = C_L\sin\alpha - C_D\cos\alpha$$

The blade airfoil section lift and drag coefficients are obtained by interpolating the available test data using both the local Reynolds number ($R_{eb}$ = $Wc/\nu$) and the local angle of attack.

In the presence of turbulence the relative velocity and the local angle of attack are given by:

$$W^2 = (\omega r - (V + u)\sin\theta - v\cos\theta)^2 + ((V + u)\cos\theta - v\sin\theta)^2\cos^2\delta \tag{21}$$

$$\alpha = \arcsin\frac{((V + u)\cos\theta - v\sin\theta)\cos\delta}{W} \tag{22}$$

The later relations are written without the subscript u because they are applied at both regions (upwind and downwind). For a given rotor geometry, rotational speed and a given velocity at each streamtube position, a value of $V_u$ is chosen assuming that the interference factor "a" is unity. Thus $Re_b$ and $\alpha$ will be evaluated in a first approximation and the airfoil characteristics $C_L$ and $C_D$ can be calculated by interpolating test data. Then, with equations (19), (20) the normal and tangential force coefficients of the blade section are estimated and equation(18) allows the upwind function $f_{up}$ to be evaluated. With the first value of $f_{up}$ another value is calculated for the interference factor employing equation (17) and the iteration process is repeated until successive sets of "a" are reasonably close. Once the true value of the induced velocity in the upwind zone has been calculated, the local relative velocity W can be obtained from equation (21) and the effective angle of attack from equation (22). The normal and tangential force are evaluated for each streamtube as a function of the blade position:

$$F_N(\theta) = (cH/S)\int_{-1}^{1} C_N(W/V_\infty)^2\ (1/\cos\delta)d\varsigma \tag{23}$$

$$F_T(\theta) = (cH/S)\int_{-1}^{1} C_T(W/V_\infty)^2\ (1/\cos\delta)d\varsigma \tag{24}$$

The torque produced by a blade element is calculated at the center of each element. By integrating along the blade we obtain the torque on a complete blade as function of the azimuthal angle:

$$T(\theta) = (\rho_\infty cRH/2)\int_{-1}^{1} C_T\ W^2(\eta/\cos\delta)d\varsigma \tag{25}$$

A similar development then leads to expressions for downstream region.

## 4. Application to Sandia 17-m wind turbine

The computer code developed for the present study is called "CARDAAS". It refers to the use of stochastic wind in the aerodynamic model. Normal forces and aerodynamic torque were calculated for Sandia 17-m wind turbine at two tip speed ratios 4.60, 2.87 and two rotational speeds 38.7 and 50.6. Results were compared to the available experimental data as given by Refs. [7], [8] and to CARDAAV predictions assuming a constant steady wind.

Figure 1 shows the distribution of the normal force coefficient as a function of the azimuthal angle for TSR = 4.60. Comparison of CARDAAS predictions, with different turbulence intensities, with experimental data and CARDAAV results show good agreements with respect to the experimental data. For high turbulence intensities the experimental data are well predicted.

At low tip speed ratio (TSR = 2.87), Fig. 2, comparison of the rotor torque predictions given by CARDAAS, CARDAAV and experimental data at three turbulence intensity levels including dynamic stall is made [9]. Results given by CARDAAS show a good agreements with respect to experimental data and deviation between CARDAAS and CARDAAV becomes greater when turbulence intensity increases.

## 5. Conclusions

A stochastic wind simulation for Darrieus wind turbine has been used to yield turbulent wind velocity fluctuations for rotationally sampled points and incorporated into the double-multiple streamtube aerodynamic model (DMS) in order to predict more accurately aerodynamic loads on wind turbine. Deterministic flow velocities from the DMS were mixed with the stochastic flow velocities which were computed from the stochastic simulation and used, afterwards, to predict forces on the rotor blades.

The average distributions of the rotor force using a turbulent wind do not coincide with the mean distributions produced by the DMS model when the constant steady wind is assumed:

- At low turbulence intensities, the deviation between the averaged values and periodic force distributions is small. Nevertheless, the instantaneous values exhibit a significant high excursions about their average.
- At high turbulence intensities, the discrepancy between the averaged and periodic force distributions increases, as does the magnitude of the instantaneous excursion.
- Normal force and rotor torque distribution agree well with experimental data and tend to be closer to them when turbulence intensity increases.

## References

[1] Thresher, R. W., Holley, W. E., Hershberg, E. L., and Lin, S. R., "Response of the MOD-DA Wind Turbine Rotor to Turbulent Atmospheric Winds", Oregon State University, Dept. of Engineering and U.S. Dept. of Energy Report DDE/RL/10378-82/1, October 1983.

[2] Strickland, J. H., "VAWT Stochastic Wind Simulator", SAND 87-0501, Sandia National Laboratories, Albuquerque, NM, April 14, 1987.

[3] Frost, W., Long, B. H., and Turner, R. E., "Engineering Handbook on the Atmospheric Environmental Guidelines for Use in Wind Turbine Generator Development", NASA Technical Paper 1359, 1978.

[4] Paraschivoiu, I., "Aerodynamic Models and Experiments for Studying Darrieus Wind Turbines", European Community Wind Energy Conference and Exhibition, Herning, Denmark, 6-10, June 1988.

[5] Strickland, J. H., "A Review of Aerodynamic Analysis Methods for Vertical Axis Wind Turbine", Proceeding of the 5th ASME Wind Energy Symposium. New Orleans, LA, February 23-24, 1986.

[6] Paraschivoiu. I., "Aerodynamic Loads and Performance of the Darrieus Rotor", Journal of Energy, Vol. 6, Nov.-Dec., 1982, pp. 406-412.

[7] Akins, R. E., "17-M Transient Pressure Measurements", 7th Annual Vertical Axis Wind Turbine Aerodynamics Seminar, Bushland, Texas, April 23-24, 1987

[8] Akins, R. E., Berg, D. E., Cyrus, W. T., "Measurements and Calculations of Aerodynamic Torques for a Vertical Axis Wind Turbine", Sandia National Laboratories SAND86-2164, Albuquerque, NM, 1987.

[9] Gormont, R. E., "A Mathematical Model of Unsteady Aerodynamics and Radial Flow Application to Helicopter Rotor", USAAMRDL, TR-72-67, May 1973.

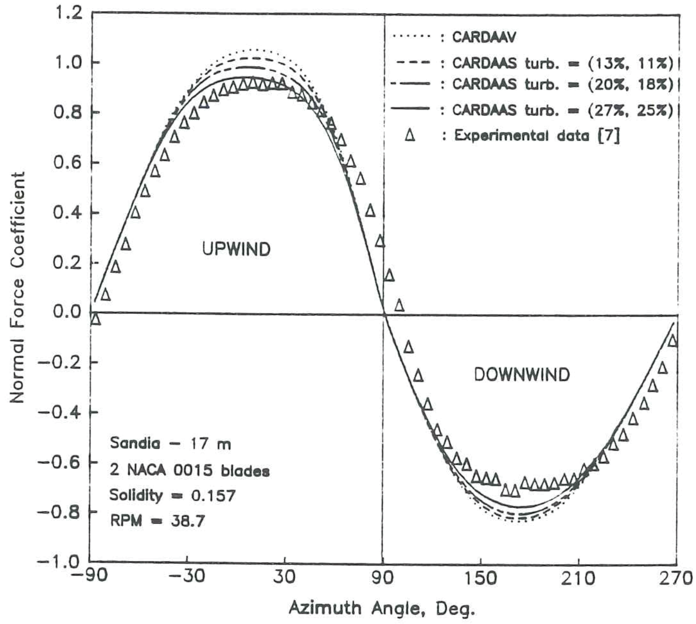

Fig. 1: Comparison of normal force coefficient at three intensity levels with experimental data and CARDAAV results at TSR = 4.60

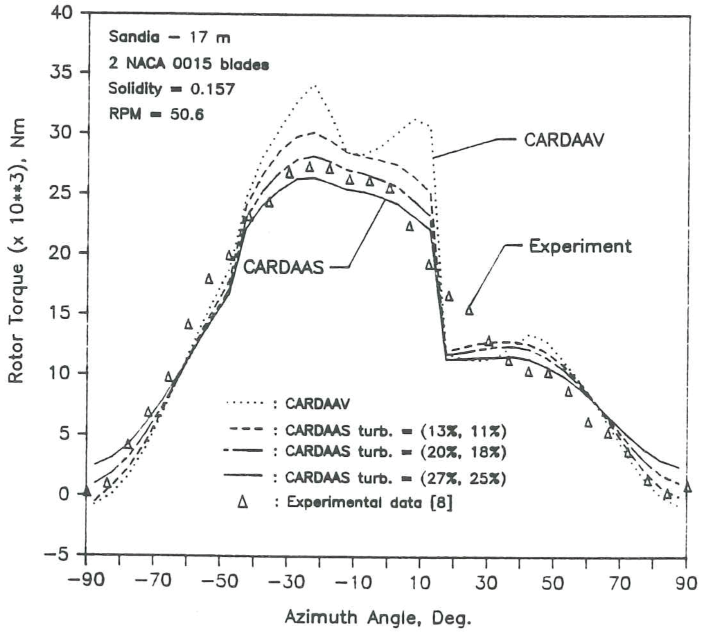

Fig. 2: Comparison of the rotor torque distribution at three intensity levels with experimental data and CARDAAV results at TSR = 2.87