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THE COMPENSATORY BARGAINING SET OF A BIG BOSS GAME

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The bargaining sets have been introduced as concepts of solution for cooperative n-person games with side payments by R.J.Aumann and M. Maschler (1964) and studied further by many authors. A comparison of various solutions for games with coalition structures has been done by R.J.Aumann and J.Dreze (1974). In a previous paper of one of the authors I.Dragan (1988), a modified concept of bargaining set, called the compensatory bargaining set, has been introduced and a combinatorial characterization of the noncore elements in the compensatory bargaining set has been obtained. This characterization becomes particularly simple for some classes of games. One of them, the class of big boss games, is considered in this paper.

The big boss games have been introduced by S.Muto, M.Nakayama, J.Potters and S.H.Tijs (1987); the bargaining set and the kernel of big boss games have been studied by J.Potters, S.Muto and S.H.Tijs (1988). In the last paper it has been shown that in the bargaining set $M_1^{(i)}$ of a big boss game there are only core elements. The present paper presents in the first section the definition of the compensatory bargaining set, to make the paper self contained; in the second section the characterization of the noncore elements in the compensatory bargaining set of a big boss game is derived (Theorem 2.5), without any use of the general characterization mentioned above. The case of the 3-person big bossgames is considered in the last section, where the set of coalitionally rational noncore elements in the compensatory bargaining set is determined (Theorem 3.1). The last result is motivating the entire study by showing noncore payoffs enjoying some type of stability.

1. The compensatory bargaining set.

Let $G = (N,v)$ be a cooperative n-person game in coalitional form. N is the set of players, $|N| = n$, and any $S \subseteq N$, $S \neq \emptyset$, is called a coalition. The characteristic function v is a real function defined on $P(N)$, the set of subsets of N, and subject to $v(\emptyset) = o$. Any partition $\mathcal{S} = (S_1,\ldots,S_p)$ of N is called a coalition structure in G. The set of admissible payoff vectors for $\mathcal{S}$ is

$$F_{\mathcal{S}} = \{ x \mid x \in R^n, \ x(S) = v(S), \forall S \in \mathcal{S} \}, \qquad (1.1)$$

where $x(S) = \sum_{i \in S} x_i$. The set of admissible payoff vectors for G is F, the union of all $F_{\mathcal{S}}$ for all coalition structures $\mathcal{S}$. The triplet (N,v,F) is a constrained game in the sense of Aumann/Dreze (1974). The core of such a game is

$$Co(N,v,F) = \{ x \mid x \in F, \ e(S,x) \leq o, \ \forall S \subseteq N \}, \qquad (1.2)$$

where for every $x \in F$ the excess function is defined by

$$e(S,x) = v(S) - x(S), \quad \forall S \subseteq N. \qquad (1.3)$$

The compensatory bargaining set of G will be a subset of F consisting of all "stable" payoffs. To define the stability principle we need some results.

Lemma 1.1: Given a coalition structure $\mathcal{S}$ and a payoff $x \in F_{\mathcal{S}}$, a coalition $C \notin \mathcal{S}$ can provide better payoffs than in x for all its members if and only if $e(C,x) > o$.

Proof: If C can provide better payoffs than in x, there is an $y \in R^c$, $c = |C|$, such that $y(C) = v(C)$ and $y_i > x_i$, $\forall i \in C$. Then, $e(C,x) > o$ follows. Conversely, if $e(C,x) > o$, then we can define $y \in R^c$ by

$$y_i = x_i + e(C,x)/|C|, \ \forall i \in C \qquad (1.4)$$

and from (1.4) we have $y_i > x_i$, $\forall i \in C$, as well as $y(C) = v(C)$.

Definition 1.2: A bargaining proposal against $x \in F_{\mathcal{S}}$ is any partial coalition structure $\mathcal{C} = (C_1,\ldots,C_k)$ with $e(C_j,x) > o$, $j=1,\ldots,k$. A partial coalition structure of G is a set of nonempty pairwise disjoint subsets of N, which may cover N or not.

Lemma 1.3: Given a coalition structure $\mathcal{S}$ and a payoff $x \in F_{\mathcal{S}}$, there is a bargaining proposal against x if and only if $x \notin Co(N,v,F)$.

Proof: The result follows from Lemma 1.1 and Definiton 1.2.

Now, Lemma 1.3 explains why any core payoff will be considered in the following as a stable payoff; it remains to explain when a noncore payoff is stable.

Let $\mathcal{C} = (C_1,\ldots,C_k)$ be a bargaining proposal against $x \in F_{\mathcal{S}}$, $x \notin Co(N,v,F)$ and denote $C_o = N - \bigcup_{j=1}^{k} C_j$, $m = |N - C_o|$; let K_j be a nonempty subset of C_j, $j=1,\ldots,k$ and $K = \bigcup_{j=1}^{k} K_j$.

Definition 1.4: Any $y \in R^m$ such that

$$y(C_j) = v(C_j), \quad j=1,\ldots,k, \quad y_i > x_i, \ i \in K, \quad y_i \geq x_i, \ i \in N-C_o \ (1.5)$$

is a bargaining distribution for $\mathcal{C}$, profitable for K.

Note that there is a bargaining distribution for $\mathcal{C}$, profitable for some K, i.e. the system (1.5) is consistent for some K, if and only if $\mathcal{C}$ is a bargaining proposal against $x \in F_{\mathcal{J}}$, $x \notin Co(N,v,F)$. The proof goes like in Lemma 1.1 with a slight change in (1.4), namely, for every C_j and $K_j \subseteq C_j$, $K_j \neq \emptyset$, we define

$$y_i = x_i + e(C_j,x)/|K_j| \ , \quad \forall\, i \in K_j \qquad (1.6)$$
$$y_i = x_i \qquad\qquad , \quad \forall\, i \in C_j - K_j .$$

Note also that if $\mathcal{C}$ is a bargaining proposal against $x \in F_{\mathcal{J}}$, $x \notin Co(N,v,F)$, then the set of bargaining distributions for $\mathcal{C}$, profitable for various K, is an infinite set $BD(\mathcal{C},x)$.

A pair $(y;\mathcal{C})$ consisting of a bargaining proposal $\mathcal{C}$ and a bargaining distribution y for $\mathcal{C}$, profitable for K, may be called a multi-objection of K against $(x;\mathcal{J})$, because it is a slight extension of the objections suggested by Aumann/Maschler (1964), section 10.

In words, a bargaining proposal is a partial coalition structure that can provide better payoffs than in x for some (or all) players belonging to each of the coalitions C_j, j=1,...,k. The distribution of each excess $e(C_j,x)$ to the members of C_j depends on the general interest of the players in C_j to make their coalition "viable".

In our model of compensatory bargaining set, a bargaining proposal $\mathcal{C}$ against x includes also a commitment: "no player in $\mathcal{C}$ has the freedom of leaving $\mathcal{C}$, unless there are groups of players from each C_j willing (and being able) to pay the compensation $e(C_j,x)$ to C_j". To counter $\mathcal{C}$ another partial coalition structure $\mathcal{D}$ should be able: first, to pay the outcomes offered in x to all players in $\mathcal{D}$; secondly, to pay the compensations $e(C_j,x)$ to C_j and, perhaps to offer something more. We define now carefully what will be called a compensatory bargaining counter proposal.

<u>Definition</u> 1.5: Two partial coalition structures $\mathcal{D} = (D_1,\ldots,D_q)$ and $\mathcal{C} = (C_1,\ldots,C_k)$ are "crossing" one the other, if

$$C_j \cap \left(\bigcup_{h=1}^{q} D_h\right) \neq \emptyset \ , \qquad j=1,\ldots,k \qquad (1.7)$$
$$D_h \cap \left(\bigcup_{j=1}^{k} C_j\right) \neq \emptyset \ , \qquad h=1,\ldots,q.$$

Note that there may be pairs D_h , C_j such that $D_h \cap C_j = \emptyset$, even though (1.7) holds.

Consider $\mathcal{D} = (D_1,\ldots,D_q)$ a partial coalition structure crossing a bargaining proposal $\mathcal{C}$ against $x \in F_{\mathcal{J}}$, $x \notin Co(N,v,F)$, and suppose $\mathcal{D}$ different of $\mathcal{C}$, i.e. no coalition in $\mathcal{D}$ is also in $\mathcal{C}$.

Denote $D_o = N - \bigcup_{h=1}^{q} D_h$, $m' = |N - D_o|$, and let $z \in R^{m'}$ be a payoff vector for players in $\mathcal{D}$, i.e. we must have

$$z(D_h) = v(D_h), \quad h=1,\ldots,q \tag{1.8}$$

Now, such a payoff takes care of the obligation of paying all players in $\mathcal{D}$ at least as much as in x, if

$$z_i \geq x_i , \quad \forall i \in N - D_o . \tag{1.9}$$

Further, $\mathcal{D}$ will be able to compensate $\mathcal{C}$, if

$$\sum_{i \in C_j \cap (N - D_o)} (z_i - x_i) \geq e(C_j,x), \quad j=1,\ldots,k. \tag{1.10}$$

Obviously, if we denote $\beta_i = z_i - x_i, \forall i \in N - D_o$, then $\mathcal{D}$ could counter $\mathcal{C}$, if the following system derived from (1.8), (1.9) and (1.10) is consistent

$$\begin{aligned} \sum_{i \in D_h} \beta_i &= e(D_h,x), & h&=1,\ldots,q, \\ \beta_i &\geq 0, & i &\in N - D_o, \\ \sum_{i \in C_j \cap (N-D_o)} \beta_i &\geq e(C_j,x), & j&=1,\ldots,k. \end{aligned} \tag{1.11}$$

In this case, $\mathcal{D}$ can pay $z_i = x_i + \beta_i$, $i \in N - D_o$.

<u>Definition</u> 1.6: A partial coalition structure $\mathcal{D}$, crossing a bargaining proposal $\mathcal{C}$ against $x \in F_{\mathcal{J}}$, $x \notin Co(N,v,F)$, and different of $\mathcal{C}$, is a compensatory bargaining counter proposal against $\mathcal{C}$, if (1.11) is consistent. Any $z \in R^{m'}$ defined by $z_i = x_i + \beta_i$, $i \in N-D_o$, where $\beta \in R^{m'}$ is a solution of (1.11), is a compensatory bargaining counter distribution for $\mathcal{D}$.

Note that $\mathcal{D}$ depends on x and $\mathcal{C}$, but does not depend on the bargaining distribution that $\mathcal{C}$ intends to make for its players. Hence, either a compensatory bargaining counter proposal against $\mathcal{C}$ does not exist, or else a counter proposal is countering simultaneously all bargaining distributions for $\mathcal{C}$. Note also that in the last case, whatever bargaining distribution for $\mathcal{C}$ is chosen, $\mathcal{D}$ is able to give to all former members of $\mathcal{C}$ their share in $\mathcal{C}$ (those members are getting also compensations).

Now, the stability principle is introduced as follows:

<u>Definition</u> 1.7: An admissible payoff x for a coalition structure $\mathcal{J}$ is compensatory stable (c-stable), if

either $x \in Co(N,v,F)$,

or, for every bargaining proposal $\mathcal{C}$ against x , there is compensatory bargaining counter proposal $\mathcal{D}$ against $\mathcal{C}$.

<u>Definition</u> 1.8: The compensatory bargaining set, M_c , is the set of all c-stable payoffs.

Obviously, we have $Co(N,v,F) \subseteq M_c$, hence if $Co(N,v,F) \neq \emptyset$

then $M_c \neq \emptyset$. At least two existence problems could be stated: first, for a given coalition structure $\mathcal{J}$, do exist $x \in F_{\mathcal{J}}$ which belong to M_c ? second, for a given coalition structure $\mathcal{J}$, do exist $x \in F_{\mathcal{J}}$ which belong to M_c - Co(N,v,F) ? In the last section of the present paper we shall show that at least for some games and some coalition structures we can give a positive answer. This will be done by using a combinatorial characterization of the noncore elements in the compensatory bargaining set. In general, there are a lot of conditions to be checked, but for some classes of games a practical tool can be derived from the general result. In the second section, we shall consider such a class, the class of big boss games, but the characterization of the noncore elements in the compensatory bargaining set will be derived only from the definitions of the first section, when we confine ourselves only to admissible individually rational payoffs.

2. Bargaining proposals and compensatory bargaining counter proposals for big boss games.

A game G = (N,v) is a big boss game, if

(a) G is monotonic, i.e. $S \subseteq T \implies v(S) \leq v(T)$;

(b) there is $i_o \in N$, called the big boss, such that

$$i_o \notin S \implies v(S) = o;$$

(c) for every S such that $i_o \in S$, we have

$$v(N) - v(S) \geq \sum_{i \notin S} [v(N) - v(N-\{i\})] . \qquad (2.1)$$

For example, in a 3-person big boss game, if we take $i_o=1$, then to satisfy (b) we must have $v(2) = v(3) = v(23) = o$. To satisfy (a), when we denote $v(1) = h$, $v(12) = a$, $v(13) = c$, $v(123) = d$, we must have

$$o \leq h \leq \min(a,c) \leq \max(a,c) \leq d. \qquad (2.2)$$

To satisfy (c), we have only one condition, namely for $S = \{1\}$, which is

$$a + c \geq d + h . \qquad (2.3)$$

We have proved a result to be used in the next section:

Lemma 2.1: A 3-person big boss game is a game

$$v(1) = h, \quad v(12) = a, \quad v(13) = c, \quad v(123) = d, \quad v(2)=v(3)=v(23)=o$$

in which

$$o \leq h \leq \min(a,c) \leq \max(a,c) \leq d , \quad a + c \geq d + h . \qquad (2.4)$$

Now, we intend to give a characterization of the admissible individually rational payoffs in M_c - Co(N,v,F) for a big boss game. Previous lemmas are needed.

Lemma 2.2: In a big boss game G, let x be an admissible individually rational payoff for a coalition structure $\mathcal{J}$. If for some coalition S we have $e(S,x) > o$, then $i_o \in S$.

Proof: If $i_o \notin S$, then $e(S,x) = -x(S)$ and because x is individually rational and G is monotonic, from $x_i \geqslant o$, $\forall i \in N$, we get $e(S,x) \leqslant o$.

Lemma 2.3: In a big boss game G, let x be an admissible individually rational payoff for a coalition structure $\mathcal{F}$ and $x \notin Co(N,v,F)$. Then, any bargaining proposal $\mathcal{C}$ against x and any compensatory bargaining counter proposal $\mathcal{D}$ against $\mathcal{C}$ should each consist of exactly one coalition which contains i_o.

Proof: Every coalition in a bargaining proposal should have a positive excess, hence by Lemma 2.2 we have $\mathcal{C} = (C)$ with $i_o \in C$. Now, if $\mathcal{D}$ is a partial coalition structure crossing $\mathcal{C}$ and different of $\mathcal{C}$, the system (1.11) could not be consistent when there is $D_h \in \mathcal{D}$ with $e(D_h,x) \leqslant o$. Hence, if $\mathcal{D}$ is a compensatory bargaining counter proposal against $\mathcal{C}$, then every coalition in $\mathcal{D}$ must have a positive excess. So, by Lemma 2.2, we have $\mathcal{D} = (D)$ with $i_o \in D$. Obviously, $C \cap D \neq \emptyset$, so that $\mathcal{D}$ and $\mathcal{C}$ are crossing one the other.

Lemma 2.3 says that in a big boss game, if $x \in F_{\mathcal{F}}$, $x \notin Co(N,v,F)$ and x is individually rational, then there is no multicoalitional bargaining proposal against x and for every bargaining proposal against x there is no multicoalitional compensatory bargaining counter proposal. In our search for c-stability we can confine ourselves to pairs of coalitions C, D with positive excesses and containing i_o.

Lemma 2.4: In a big boss game G, let x be an individually rational admissible payoff for a coalition structure $\mathcal{F}$. If $\mathcal{C} = (C)$ with $i_o \in C$ is a bargaining proposal against x, i.e. $e(C,x) > o$, then $\mathcal{D} = (D)$ with $i_o \in D$, $D \neq C$, is a compensatory bargaining counter proposal against $\mathcal{C}$, if and only if

$$e(D,x) \geqslant e(C,x) \ . \tag{2.5}$$

Proof: Under the assumptions, $\mathcal{D} = (D)$ is a compensatory bargaining counter proposal against $\mathcal{C}$, if and only if the following system derived from (1.11) is consistent:

$$\sum_{i \in D} \beta_i = e(D,x) \ , \quad \beta_i \geqslant o \ , \ \forall i \in D \ , \qquad \sum_{i \in C \cap D} \beta_i \geqslant e(C,x) \ . \tag{2.6}$$

Now, if (2.6) is consistent, then we have

$$e(D,x) = \sum_{i \in D} \beta_i = \sum_{i \in C \cap D} \beta_i + \sum_{i \in D-C} \beta_i \geqslant e(C,x) \tag{2.7}$$

i.e. (2.5) holds. Conversely, if (2.5) holds we may define

$$\beta_i = \begin{cases} e(C,x)/|C \cap D| & \text{if } i \in C \cap D \\ [e(D,x) - e(C,x)]/|D-C| & \text{if } i \in D-C \text{ and } D-C \neq \emptyset \end{cases} \quad (2.8)$$

Obviously, (2.8) gives a solution of (2.6), which is consistent, hence $\mathcal{D} = (D)$ is a compensatory bargaining counter proposal against $\mathcal{C}$.

Now, Lemmas 2.3 and 2.4 are helpful in proving the following:

Theorem 2.5: In a big boss game G, let x be an individually rational admissible payoff for a coalition structure $\mathcal{S}$. Then, we have $x \in M_c - Co(N,v,F)$ if and only if

$$H = \max_S e(S,x) > o \quad (2.9)$$

and H is reached for at least two coalitions which contain i_o.

Proof: If $x \in M_c - Co(N,v,F)$, then there is at least one coalition of positive excess, so that H will be positive. If H is reached for exactly one coalition C^+, that coalition should contain i_o by Lemma 2.3. If $\mathcal{D} = (D)$ is a compensatory bargaining counter proposal against $\mathcal{C} = (C^+)$, then by Lemma 2.4 we must have $e(D,x) \geqslant e(C^+,x) = H$, which is impossible. Hence, $\mathcal{C}$ has no compensatory bargaining counter proposal and this contradicts the assumpyion $x \in M_c - Co(N,v,F)$. Conversely, if there are at least two coalitions C_1^+ and C_2^+ such that $H = e(C_1^+,x) = e(C_2^+,x) > o$, then both C_1^+ and C_2^+ can be used for bargaining proposals against x and $i_o \in C_1^+ \cap C_2^+$. By Lemma 2.4, $\mathcal{C}_1 = (C_1^+)$ is countering $\mathcal{C}_2 = (C_2^+)$ and viceversa. Moreover, any other bargaining proposal $\mathcal{C} = (C)$ is countered by $\mathcal{C}_1$ and $\mathcal{C}_2$, because $e(C_1^+,x) = e(C_2^+,x) \geqslant e(C,x)$, hence $x \in M_c - Co(N,v,F)$.

Theorem 2.5 shows that for a big boss game G, to determine whether an admissible individually rational payoff x for a coalition structure $\mathcal{S}$ belongs to $M_c - Co(N,v,F)$, the computation of H is needed (confined to coalitions which contain i_o). If $H = \max_S e(S,x)$ is not positive, then $x \in Co(N,v,F)$; if H is positive, then by Theorem 2.5, either H is reached for a unique coalition and $x \notin M_c$, or H is reached for at least two coalitions and we have $x \in M_c - Co(N,v,F)$.

Note also that Theorem 2.5 holds when the admissible payoff x is supposed to be coalitionally rational instead of individually rational. This last result will be used for finding the set of coalitionally rational admissible payoffs belonging to $M_c - Co(N,v,F)$, for a 3-person big boss game in the next section.

3. The 3-person big boss game.

Consider the 3-person big boss game G defined by

$v(1) = h$, $v(12) = a$, $v(13) = c$, $v(123) = d$, $v(2)=v(3)=v(23)=o$

where by Lemma 2.1 we have

$$o \leq h \leq \min(a,c) \leq \max(a,c) \leq d, \quad a + c \geq d + h \ . \quad (3.1)$$

We intend to determine the coalitionally rational payoffs belonging to M_c - Co(N,v,F). As these payoffs are also individually rational, we shall consider all possible coalition structures and use Theorem 2.5.

1) For $\mathcal{S}_o = (123)$, a coalitionally rational payoff $x \in F_{\mathcal{S}_o}$, if any, belongs to Co(N,v,F).

2) For $\mathcal{S}_1 = (1,2,3)$, the unique admissible payoff

$$(\sigma_1) \qquad x_1 = h, \quad x_2 = o, \quad x_3 = o \ ,$$

gives as possible positive excesses

$$e(12,x) = a - h, \quad e(13,x) = c - h, \quad e(123,x) = d - h. \quad (3.2)$$

By (3.1) and Theorem 2.5, we have $x \in M_c$ - Co(N,v,F) if and only if

$$d = a > c, \quad \text{or} \quad d = c > a, \quad \text{or} \quad d = a = c > h \ . \quad (3.3)$$

3) For $\mathcal{S}_2 = (13,2)$, the coalitionally rational payoffs should satisfy

$$x_1 - x_3 = c, \quad x_2 = o, \quad x_1 \geq h, \quad x_3 \geq o \quad (3.4)$$

and the only possible positive excesses are

$$e(12,x) = a - x_1, \quad e(123,x) = d - c \ . \quad (3.5)$$

By (3.5) and Theorem 2.5, if $x \in M_c$ - Co(N,v,F) then we should have $a - x_1 = d - c$. This equality and the admissibility give

$$(\sigma_2) \qquad x_1 = a + c - d, \quad x_2 = o, \quad x_3 = d - a \ .$$

Now, (3.4) are satisfied, because $a + c - d \geq h$ and $d - a \geq o$ follow from (3.1). Moreover, we have $e(12,x) = e(123,x) = d - c$, hence the last necessary condition, $H > o$, is $c < d$. It is easy to see that $c < d$ is also sufficient for (σ_2) to be in M_c - Co(N,v,F).

4) For $\mathcal{S}_3 = (12,3)$, a similar computation shows that

$$(\sigma_3) \qquad x_1 = a + c - d, \quad x_2 = d - c, \quad x_3 = o$$

is the unique coalitionally rational payoff admissible for $\mathcal{S}_3$, which belongs to M_c - Co(N,v,F) if and only if $a < d$.

5) For $\mathcal{S}_4 = (23,1)$, the unique admissible payoff is (σ_1) and we get the same result as for the individual play.

The considerations given above prove:

<u>Theorem</u> 3.1: In a 3-person big boss game G

$$v(1) = h, \quad v(12) = a, \quad v(13) = c, \quad v(123) = h, \quad v(2)=v(3)=v(23)=o,$$

the set of coalitionally rational admissible payoffs in M_c - Co(N,v,F) consists of the following payoffs:

(σ_1) $x_1=h$, $x_2=o$, $x_3=o$, for $\mathcal{S}_1=(1,2,3)$, if $d=a=c > h$;

then, (σ_1) for $\mathcal{S}_1=(1,2,3)$ and

(σ_2) $x_1=a+c-d$, $x_2=o$, $x_3=d-a$ for $\mathcal{S}_2=(13,2)$, if $d=a>c$;
then, (σ_1) for $\mathcal{S}_1=(1,2,3)$ and
(σ_3) $x_1=a+c-d$, $x_2=d-c$, $x_3=o$ for $\mathcal{S}_3=(12,3)$, if $d=c>a$;
then
(σ_2) for $\mathcal{S}_2$ and (σ_3) for $\mathcal{S}_3$, if $d>\max(a,c)$.

Note that in the proof of Theorem 3.1, beside the individual rationality conditions $x_1 \geqslant h$, $x_2 \geqslant o$, $x_3 \geqslant o$, the other conditions included in the coalitional rationality conditions have been used only in the search for c-stable payoffs for $\mathcal{S}_o=(123)$. Therefore, for all the other coalition structures the results stated in Theorem 3.1 are still holding when the words "coalitionally rational admissible payoffs" are replaced by "individually rational admissible payoffs". However, there is a family of individually rational admissible payoffs for $\mathcal{S}_o=(123)$ which is c-stable and should be added to the new statement.

Theorem 3.1 is showing noncore payoffs belonging to tne compensatory bargaining set of a game, hence this concept of solution is meaningfull. Of course, the existence problems stated above remain open.

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