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Null Controllability of Semilinear Stochastic Systems Governed by B-Evolutions on Hilbert Spaces

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Abstract. Null controllability of semilinear stochastic systems governed by B-evolutions on Hilbert spaces is studied by using a well-known fixed point theorem. An application to stochastic partial differential equations is given.

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1 Introduction

Semigroup theory gives a unified treatment of a wide class of stochastic parabolic, hyperbolic and functional differential equations, and much effort has been devoted to the study of controllability results for such evolution equations (see [10], [11]). In deterministic cases, fixed point techniques, among the other methods, are widely used as a tool for studying the controllability of nonlinear systems. Several authors have extended finite dimensional controllability results to infinite dimensional controllability results represented by evolution equations with bounded and unbounded linear operators in Banach spaces (for example, see Balachandran *et. al.* [4] and Dauer *et. al.* [8]). Stochastic control theory is a stochastic generalization of classical control theory. Controllability of linear stochastic systems is a well-known problem discussed in the literature ([3], [10], [13], [14]). Controllability of a linear stochastic system in Hilbert space recently has been extended by Balasubramaniam [5] to quasilinear stochastic evolution equation using a fixed point approach. The purpose of this paper is to consider the controllability of semilinear stochastic systems governed by B-evolution on Hilbert spaces. The considered system is an abstract formulation of many stochastic partial differential equations. For motivation, the heat transfer equation is discussed in the final section.

The main objective of this paper is to derive controllability conditions for the following semilinear stochastic B-evolution equation

$$\frac{dBx(t)}{dt} = Ax(t) + (Cu)(t) + f(Bx(t)) + g(Bx(t))\frac{dw(t)}{dt}, \quad t \in J = [0, T] \tag{1}$$

$$Bx|_{t=0} = y_0, \tag{2}$$

where $< A, B >$ is the generating pair of the B-evolution $S(t),\ t > 0$. The state $x(\cdot)$ takes its values in the Hilbert space H. The control function $u(\cdot)$ is given in $L^2(J, U)$, a Hilbert space of admissible control functions with U be a Hilbert space and let C be a bounded linear operator from U into H.

Let $(\Omega, F, F_t, \mathbf{P})$ be a complete probability space furnished with complete family of right continuous increasing sigma algebras $\{F_t, t \in J\}$ satisfying $F_t \subset F$. Let H be a separable Hilbert space, and let $\{w(t), t \geq 0\}$ be a Brownian motion with values in H having mean zero and covariance operator Q. Then

$$E\{(w(t), h_1)(w(t), h_2)\} = t(Qh_1, h_2) \text{ forevery } h_1, h_2 \in H.$$

Here the standard notation $E\varphi = \int_\Omega \varphi(\omega)\mathbf{dP}$ is used to denote the integration of any F-measurable function φ with respect to the probability measure $\mathbf{P}$.

Assume that f and g are time invariant continuous mappings defined on appropriate spaces. For any Hilbert space K and any $t \geq 0$, let $L^2(F_t, K, \mathbf{P})$ denote the space of F_t -measurable K-valued random variables which are square integrable in the Bochner sense with respect to the $\mathbf{P}$ measure. Clearly this is a family of Hilbert spaces with natural scalar product.

2 Preliminaries

In this section, for convenience, some facts from the theory of B-evolutions are given. (For more details, see ([6], [7], [12]). Let X and Y be two separable Hilbert spaces with scalar products $< \cdot, \cdot >_X$, $< \cdot, \cdot >_Y$, and denote the norms by $\| \cdot \|_X$ and $\| \cdot \|_Y$, respectively. Let B be a linear operator with domain $D(B) \subset X$ and range $R(B) \subset Y$ and denote $< A, B >$ as the generating pair of the B-evolution $S(t),\ t > 0$. Assume that B is either closed or closeable, with its closed extension again denoted by B. Introduce the vector space $X_B = \{x \in X : \|x\|_X < \infty, \|Bx\|_Y < \infty\}$, furnished with the topology associated graph norm defined by

$$< \xi, \zeta >_{X_B} =< \xi, \zeta >_X + < B\xi, B\zeta >_Y, \quad \text{for} \ \ \xi, \zeta \in X_B,$$

$$\|x\|_{X_B} = \|x\|_X^2 + \|Bx\|_Y^2)^{(1/2)}, \quad \text{for} \ \ x \in X_B.$$

Since B is closed, X_B is a Banach space with respect to this norm topology.

Let $M_2(J, X_B)$ denote the equivalence classes of X_B-valued stochastic processes $\{x(t) : t \in J\}$ which are F_t-adapted and have finite second moments, that is,

$$\|x\|_{M_2} = (\sup_{t \in J} E\|x(t)\|_{X_B}^2)^{(1/2)} < \infty.$$

The space M_2 furnished with the above norm topology is a Banach space.

Definition 2.1. A family of bounded linear operators $\{S(t), t > 0\}$ defined on Y is called a *B-evolution* if $S(t)(Y) \subset D(B)$ for all t > 0 and

$$S(t + s) = S(s)BS(t), \quad \text{for all} \ \ t, s > 0.$$

Associated with any B-evolution $S(t)$, $t > 0$, there is a semigroup of bounded linear operators $\{R(t), t > 0\}$ in Y given by

$$R(t) \equiv BS(t), \quad t > 0.$$

The B-evolution property can also be expressed in terms of the semigroup as follows

$$S(t+s) = S(t)R(s) = S(s)R(t), \quad t, s > 0.$$

It is clear from this fact that $R(t+s) = R(t)R(s)$ for $t, s > 0$. The B-evolution $S(t)$ is said to be *strongly continuous* if $\{R(t), t > 0\}$ is a C_0- semigroup in Y.

Definition 2.2. The infinitesimal generator A of a B-evolution $S(t)$ is given by

$$D(A) = \{x \in D(B) : \lim_{r \to 0} A_r x \ \text{ exists}\},$$
$$Ax \equiv \lim_{r \to 0} A_r x, \quad \text{for} \ \ x \in D(A).$$

It is clear from the definition of the infinitesimal generator A that $D(A) \subset D(B)$, (for more details see Sauer [12]).

Lemma 2.3. ([12], p.290) *Let $S(t)$, $t > 0$, be a strongly continuous uniformly bounded B-evolution. Then*

a) *for each $y \in Y$, the map $t \to S(t)y_0$, $t > 0$, is strongly continuous with values in X;*
b) *there exists an operator $C \in L(Y, X)$ such that $Cy_0 = \lim_{t \to 0} S(t)y_0$ for each $y_0 \in Y$ and $S(t)y_0 = CR(t)y_0$, $t > 0$;*
c) *C restricted to the range of the operator B, $R(B)$, is the right inverse of B.*

To derive controllability conditions for semilinear stochastic systems, assume the following hypotheses.

(i) The pair $< A, B >$ is the generating pair of a uniformly bounded B-evolution $S(t)$. The B-evolution $S(t)$, $t > 0$, and the associated C_0- semigroup $R(t)$, $t > 0$, are uniformly bounded by positive constants N and M respectively, with

$$\sup_{t \in J} \|S(t)\|^2_{L(Y, X_B)} \leq (N^2 + M^2) \equiv N_1$$

for some positive constant N_1.

(ii) w is an H-valued Wiener process defined on a Hilbert space K.

(iii) The maps $f_B : X_B \to Y$ and $g_B : X_B \to L(H, Y)$ are continuous and there exists a constant $\alpha > 0$ such that for all $\xi, \zeta \in X_B$,

$$\|f_B(\zeta)\|^2_Y \vee \|g_B(\zeta)\|^2_{L(H,Y)} \leq \alpha^2(1 + \|\zeta\|^2_{X_B}),$$
$$\|f_B(\xi) - f_B(\zeta)\|^2_Y \vee \|g_B(\xi) - g_B(\zeta)\|^2_{L(H,Y)} \leq \alpha^2 \|\xi - \zeta\|^2_{X_B},$$

where $f_B(x) \equiv (f \circ B)(x)$, $g_B(x) \equiv (g \circ B)(x)$ are the composition operators.

(iv) The covariance operator $Q \in L(H)$ is positive nuclear with eigenvectors $\{e_i\}$ corresponding to the eigenvalues $\{\lambda_i\}$; so that $Qe_i = \lambda_i e_i$ and $\{e_i\}$ is a complete orthonormal basis of H.

(v) The linear operator Z from $L^2(J,U)$ into H defined by

$$Zu = \int_0^T S(T-s)Cu(s)ds$$

has an invertible operator Z^{-1} defined on $L^2(J,U) \setminus KerZ$, (see [9]), and there exist positive constants N_2, N_3 such that

$$\|C\|^2 \le N_2 \text{ and } \|Z^{-1}\|^2 \le N_3.$$

If the conditions (i)-(iv) are satisfied, then for every $y_0 \in L^2(F_0, Y, \mathbf{P})$ there is an element $x \in M_2(J, X_B)$, said to be a mild solution of equations (1)–(2), given by (see [1])

$$x(t) = S(t)y_0 + \int_0^t S(t-s)(Cu)(s)ds + \int_0^t S(t-s)f_B(x(s))ds + \int_0^t S(t-s)g_B(x(s))dw(s), \tag{3}$$

Definition 2.4. ([10]) The stochastic system (1) - (2) is said to be *null controllable* on J, if for some control $u \in L^2(J,U)$, the solution of (1) - (2) with $y_0 \in L^2(F_0, Y, \mathbf{P})$ is such that $x(T) = 0$, where 0 and T are preassigned terminal state and time, respectively. If the system is null controllable for all y_0, it is called *completely null controllable* on J.

3 Main Result

Theorem 3.1. *Suppose the hypotheses (i) - (v) are satisfied, then the system (1) - (2) is completely null controllable on J.*

Proof. Using the hypothesis (v), define the control

$$u(t) = -Z^{-1}\Big[S(T)y_0 + \int_0^T S(T-s)f_B(x(s))ds + \int_0^T S(T-s)g_B(x(s))dw(s)\Big](t).$$

Now it is shown that when using this control the operator defined by

$$\begin{aligned}(\Phi x)(t) = S(t)y_0 - \int_0^t S(t-\eta)CZ^{-1}\Big[S(T)y_0 + \int_0^T S(T-s)f_B(x(s))ds \\ + \int_0^T S(T-s)g_B(x(s))dw(s)\Big](\eta)d\eta \\ + \int_0^t S(t-s)f_B(x(s))ds + \int_0^t S(t-s)g_B(x(s))dw(s)\end{aligned}$$

has a fixed point. This fixed point is a solution of equation (1)–(2). Clearly

$$(\Phi x)(t)|_{t=0} = y_0,$$

which means that the control $u(\cdot)$ steers the semilinear evolution equation from the initial state y_0 to 0 in time T provided the nonlinear operator Φ has a fixed point.

First, it is shown that Φ maps $M_2(J, X_B)$ into itself. Note that by virtue of assumption (iv), the Brownian motion w has the representation

$$w(t) = \sum_{i=1}^{\infty} (\lambda_i)^{(1/2)} \beta_i(t) e_i,$$

where $\{\beta_i\}$ are real-valued standard F_t-Brownian motions. Further

$$|a+b+c|^2 \le 9(|a|^2+|b|^2+|c|^2)$$

for any real numbers a, b, c. Using assumptions (iii), (iv) and the fact that $y_0 \in L^2(F_0, Y, \mathbf{P})$, it follows from direct computation that, for $t \in J$,

$$\begin{aligned}
\|(\Phi x)(t)\|_{M_2}^2 \le 9\Big\{ &\sup_{t\in J} E(\|S(t)y_0\|_Y^2) + E\Big\| \int_0^t S(t-\eta)BZ^{-1}\Big[S(T)y_0 \\
&+ \int_0^T S(T-s)f_B(x(s))ds + \int_0^T S(T-s)g_B(x(s))dw(s)\Big](\eta)d\eta\Big\|_{X_B}^2\Big\} \\
&+ 9\Big\| \int_0^t S(t-s)f_B(x(s))ds\Big\|_{X_B}^2 + 9\Big\|\int_0^t S(t-s)g_B(x(s))dw(s)\Big\|_{X_B}^2 \\
\le 9N_1\Big\{&E\|y_0\|_Y^2 + TN_2N_3\Big[N_1E\|y_0\|_Y^2 + N_1T\alpha^2\int_0^T\Big(1+E\|x(s)\|_{X_B}^2\Big)ds \\
&+ \alpha^2 N_1 \operatorname{tr} Q \int_0^T \Big(1+E\|x(s)\|_{X_B}^2\Big)ds\Big] \\
&+ \alpha^2 T\int_0^t\Big(1+E\|x(s)\|_{X_B}^2\Big)ds + \alpha^2 \operatorname{tr} Q\int_0^t\Big(1+E\|x(s)\|_{X_B}^2\Big)ds\Big\} \\
\le 9N_1&\{1+TN_1N_2N_3\}E\|y_0\|_Y^2 + 9N_1\alpha^2(TN_1N_2N_3+1)(T+\operatorname{tr} Q) \\
&\quad\times\int_0^T\Big(1+E\|x(s)\|_{X_B}^2\Big)ds,
\end{aligned}$$

where $\operatorname{tr} Q = \sum \lambda_i$. Hence, there exists a constant $\gamma = \gamma(N_1, N_2, N_3, T, \operatorname{tr} Q, \alpha)$ dependent on the parameters as indicated such that

$$\|(\Phi x)(t)\|_{M_2}^2 \le \gamma^2\Big[E\|y_0\|_Y^2 + \int_0^T\Big(1+E\|x(s)\|_{X_B}^2\Big)ds\Big]. \tag{4}$$

Since $x \in M_2(J, X_B)$, both f_B and g_B are continuous, and y_0 is F_0- measurable, it is clear that $(\Phi x)(t)$, $t \in J$ is F_t- measurable. It follows from inequality (4) that for any $x \in M_2(J, X_B)$, $\Phi x \in M_2(J, X_B)$ proving that Φ maps M_2 into itself.

Now it is shown that $\Phi \in C(J, L^2(F, X_B))$. Let I_Y denote the identity operator in Y. For $t \in (0, T]$ and $h > 0$, we have

$$\begin{aligned}
E\Big\|(\Phi z)(t+h) - (\Phi z)(t)\Big\|_{X_B}^2 &\le 9N_1E\Big(\|(R(h)-I_Y)y_0\|_Y^2\Big) \\
&+ 18N_1N_2N_3t\int_0^t E\|(R(h)-I_Y)y_0\|_Y^2 E\Big\|\Big[S(T)y_0 + \int_0^T S(T-s)f_B(x(s))ds \\
&+ \int_0^T S(T-s)g_B(x(s))dw(s)\Big](\eta)\Big\|_{X_B}^2 d\eta
\end{aligned}$$

$$+ 18N_1N_2N_3h\int_t^{t+h} E\Big\|\Big[S(T)y_0 + \int_0^T S(T-s)f_B(x(s))ds$$

$$+ \int_0^T S(T-s)g_B(x(s))dw(s)\Big](\eta)\Big\|_{X_B}^2 d\eta$$

$$+ 18N_1t\int_0^t E\Big\|(R(h)-I_Y)f_B(x(s))\Big\|_Y^2 ds + 18N_1h\int_t^{t+h} E\|f_B(x(s))\|_Y^2 ds$$

$$+ 18N_1 \operatorname{tr} Q\int_0^t E\Big\|(R(h)-I_Y)g_B(x(s))\Big\|_{L(H,Y)}^2 ds$$

$$+ 18N_1 \operatorname{tr} Q\int_t^{t+h} E\|g_B(x(s))\|_{L(H,Y)}^2 ds.$$

Similarly, for $h > 0$ and $t - h \geq 0$

$$E\Big\|(\Phi z)(t-h) - (\Phi z)(t)\Big\|_{X_B}^2 \leq 9N_1E\Big(\|(R(h)-I_Y)y_0\|_Y^2\Big)$$

$$+ 18N_1N_2N_3(t-h)\int_0^{t-h} E\|(R(h)-I_Y)y_0\|_Y^2 E\Big\|\Big[S(T)y_0 + \int_0^T S(T-s)f_B(x(s))ds$$

$$+ \int_0^T S(T-s)g_B(x(s))dw(s)\Big](\eta)\Big\|_{X_B}^2 d\eta$$

$$+ 18N_1N_2N_3h\int_{t-h}^{t} E\Big\|\Big[S(T)y_0 + \int_0^T S(T-s)f_B(x(s))ds$$

$$+ \int_0^T S(T-s)g_B(x(s))dw(s)\Big](\eta)\Big\|_{X_B}^2 d\eta$$

$$+ 18N_1(t-h)\int_0^{t-h} E\Big\|(R(h)-I_Y)f_B(x(s))\Big\|_Y^2 ds + 18N_1h\int_{t-h}^{t} E\|f_B(x(s))\|_Y^2 ds$$

$$+ 18N_1 \operatorname{tr} Q\int_0^{t-h} E\Big\|(R(h)-I_Y)g_B(x(s))\Big\|_{L(H,Y)}^2 ds$$

$$+ 18N_1 \operatorname{tr} Q\int_{t-h}^{t} E\|g_B(x(s))\|_{L(H,Y)}^2 ds.$$

Using arguments based on Fubini's Theorem, the Lebesgue Dominated Convergence Theorem, and the C_0-property of the semigroup $\{R(t), t > 0\}$, the continuity of Φ follows from the above two inequalities. Hence $\Phi \in C(J, L^2(F, X_B))$.

It remains to show that Φ is a contraction map. Let ζ, η be any two elements of $M_2(J, X_B)$. Then from a similar computation to that given above, it is easy to verify that for $\zeta(0) = \eta(0) = y_0$,

$$\begin{aligned}
&E\Big\|(\Phi\zeta)(t)-(\Phi\eta)(t)\Big\|^2_{X_B}\\
&\qquad\le(9N_1^2N_2N_3+9N_1)(T+\operatorname{tr}Q)\int_0^T E\|\zeta(s)-\eta(s)\|^2_{X_B}ds\\
&\qquad\le\alpha\int_0^T E\|\zeta(s)-\eta(s)\|^2_{X_B}ds,
\end{aligned}\tag{5}$$

where $\alpha=(9N_1^2N_2N_3+9N_1)(T+\operatorname{tr}Q)$. By repeated substitution using (5), after n iterations the following holds

$$E\|(\Phi^n\zeta)(t)-(\Phi^n\eta)(t)\|^2_{X_B}\le\frac{\alpha^n}{(n-1)!}\int_0^T(t-s)^{n-1}E\|\zeta(s)-\eta(s)\|^2_{X_B}ds,\quad t\in J.$$

Hence

$$E\|(\Phi^n\zeta)-(\Phi^n\eta)\|^2_{M_2}\le\frac{(\alpha T)^n}{n!}\|\zeta-\eta\|^2_{M_2}.$$

This shows that, for sufficiently large n, $(\alpha T)^n/n!<1$ and Φ^n is a contraction. Hence by the Banach fixed point theorem, Φ and Φ^n have only one and the same fixed point in $M_2(J,X_B)$. Any fixed point of Φ is a solution of (1) - (2) on J satisfying $(\Phi x)(t)=x(t)\in M_2(J,X_B)$ for all y_0 and T. Thus system (1) - (2) is completely null controllable on J.

4 Example

As an application of the above results, consider the following (deterministic) heat transfer equation with dynamic boundary condition. Let $\Omega\subset R^n$, $n=1,2,3$, be an open, bounded domain with smooth boundary which consists of two parts $\partial\Omega\equiv\Gamma_0\cup\Gamma_1$. The material (e.g., fluid) in the interior of the domain receives heat energy through the boundary Γ_1 from an external source distributed on the exterior of the boundary layer Γ_1.

Taking into account the dynamics of heat source, the problem can be modelled as follows,

$$\begin{aligned}
\frac{\partial T(t,\xi)}{\partial t}&=\operatorname{div}(\sigma(\xi)\nabla T)+v.\nabla T+f(t,\xi,T(t,\xi)),\\
&\qquad\qquad t>0,\quad \xi\in\Omega,\quad T(t,\xi)|_{\Gamma_0}=0,\\
\frac{\partial T(t,\xi)}{\partial t}\bigg|_{\Gamma_1}&=-\beta D_vT(t,\xi)|_{\Gamma_1}+g(t,\xi,T(t,\xi)|_{\Gamma_1},u)\\
T(0,\xi)=T_0(\xi),&\quad \xi\in\Omega,\quad T(0,\xi)|_{\Gamma_1}=T_1(x),\quad \xi\in\Gamma_1.
\end{aligned}$$

Here T denotes the space-time temperature distribution in the interior of the domain, and $\sigma: cl\ \Omega\to[0,\infty)$ represents the thermal conductivity which equals a constant $r\ (>0)$ in Ω and $\beta\ (>0)$ on the boundary $\partial\Omega$. The constant β represents the thermal conductivity of the material that constitutes the boundary layer Γ_1. The quantity $v\equiv v(t,\xi)\in R^3$ denotes the transport velocity of the material. The function f represents the internal heat source and is possibly nonlinear. The function g represents a nonlinear heat transfer characteristic that denotes the temperature of the external source on the boundary part Γ_1. D_v denotes the outward normal derivative. This system has a dynamic boundary condition and an internal control $u \equiv u(t,\xi)$ with bounded linear control operators $C_i,\ i=1,2$. (For more examples of stochastic boundary problems see references [1], [2], [7], [12].)

More generally, consider the stochastic partial differential equation given by

$$\frac{\partial\varphi}{\partial t} = L_1\varphi + (C_1u)(t,\xi) + F_1(t,\xi,\varphi) + F_2(t,\xi,\varphi)N_d(t,\xi), \quad t \geq 0, \quad \xi \in \Omega, \tag{6}$$

$$\frac{\partial(\tau\varphi)}{\partial t} = L_2\varphi + (C_2u)(t,\xi) + G_1(t,\xi,\tau\varphi) + G_2(t,\xi,\tau\varphi)N_b(t,\xi), \quad t \geq 0, \quad \xi \in \Omega, \tag{7}$$

where Ω is an open, bounded, connected subset of R^n. The processes N_d and N_b are suitable random fields distributed in the interior and the boundary of the set Ω, respectively. The operators L_1, L_2 and τ are linear partial differential operators and F_i, G_i, $i=1,2$, are nonlinear as described below

$$\begin{aligned}
L_1\varphi &\equiv \sum_{|\beta|\leq m} a_\alpha(t,\xi)D^\alpha\varphi,\\
L_2\varphi &\equiv \sum_{|\alpha|\leq m-1} b_\beta(t,\xi)D^\beta\varphi,\\
\tau\varphi &\equiv \{\varphi|_{\partial\Omega}, D_v\varphi|_{\partial\Omega}, \ldots, D_v^{m-1}\varphi|_{\partial\Omega}\},\\
F_i(t,\xi,\varphi) &\equiv f_i(t,\xi,\varphi,D\varphi,\ldots,D_{m-1}\varphi),\ i=1,2,\\
G_i(t,\xi,\tau\varphi) &\equiv g_i(t,\xi,\varphi,D_v\varphi,\ldots,D_v^{m-1}\varphi),\ i=1,2.
\end{aligned}$$

The operator τ is the trace operator, D_v^k denotes the normal derivatives on the boundary of order exactly k, with $k \leq m-1$, and similarly D_k denotes all the spatial derivatives of order exactly k.

Using appropriate Sobolov spaces and the Hilbert spaces, $L_2(\Omega)$ and $L_2(\partial\Omega)$, system (6) - (7) can be realized as an abstract formulation of the following stochastic semilinear equations

$$\frac{d(Bx(t))}{dt} = Ax(t) + (Cu)(t) + f(t,Bx(t)) + g(t,Bx(t))\frac{dw(t)}{dt}, \quad t \in J = [0,T], \tag{8}$$

$$s - \lim_{t\to 0^+} Bx(t) = y_0 \tag{9}$$

With reference to system (6) - (7), here B, A, C, f and g represent the pair of operators $\{I,\tau\}$, $\{L_1,L_2\}$, $\{C_1,C_2\}$, $\{F_1,G_1\}$ and $\{F_2,G_2\}$, respectively, where I represents the identity operator in H, and finally the random fields are modelled as abstract Weiner processes on another Hilbert space, say H. Clearly the nonlinear operators are maps, so that $f: J\times Y \to Y$ and $g: J\times Y \to L(H,Y)$.

More concrete spaces suitable for the example given above can be specified as an account of random disturbances on the mast and antenna, a stochastic model can be formulated as follows.

Introduce the Hilbert space $H \equiv L_2(\Omega)\times R^2$ with the natural inner product. Define the bounded linear operator $C: U \to Y$ such that it satisfies hypothesis (v) and define the operator $g : Y \to L(H, Y)$ and the H-valued Brownian motion as follows

$$g(y)h \equiv \{0, g_2(y)h_1, 0, g_3(y_4)h_2, g_4(y_5)h_3\}, \quad y \in Y, \quad h \in H,$$
$$W(t) \equiv \{W_1(t, .), W_2(t), W_3(t)\},$$

where the distributional derivative of W_1 represents random forces acting on the mast (originating possibly from different sources such as micrometiorities bombarding the beam, gravity gradient, solar flares, etc.,) and those of W_2, W_3 represent shear forces and bending moments exerted at the free end of the beam (mast) by similar events. Here $\{g_2, g_3, g_4\}$ are suitable functions allowing both additive and multiplicative disturbances. The stochastic counterpart of the system (8) - (9) is then given by

$$\frac{d(Bx(t))}{dt} = Ax(t) + (Cu)(t) + f(Bx(t)) + g(Bx(t))\frac{dw(t)}{dt}, \quad t > 0, \tag{10}$$

$$Bx(t)|_{t=0} = y_0. \tag{11}$$

Under these assumptions, Theorem 3.1 applies and hence system (10) - (11) is completely null controllable.

References

[1] N. U. Ahmed, *Stochastic B-evolutions on Hilbert spaces*, Nonl. Anal., Theory, Methods and Appl., **30** (1997), 199-209.

[2] N. U. Ahmed and S. Kerbal, *Stochastic systems governed by B-evolutions on Hilbert spaces*, Proc. Roy. Soc. Edinburgh, **127A** (1997), 903-920.

[3] K. J. Astrom, *Introduction to Stochastic Control Theory*, Academic Press, New York, 1970.

[4] K. Balachandran, J. P. Dauer and P. Balasubramaniam, *Controllability of nonlinear integrodifferential systems in Banach space*, J. Optim. Theory Appl., **84** (1995), 83-91.

[5] P. Balasubramaniam, *Controllability of quasilinear stochastic evolution equations in Hilbert space*, J. Appl. Math. Stoch. Anal., **14** (2001), 151-159.

[6] M. van Dalsen, *Evolution problems involving non-stationary operators between two Banach spaces I, existence and uniqueness theorems*, Quaestiones Math., **8** (1985), 97-129.

[7] M. van Dalsen, *Semilinear evolution equations and fractional powers of a closed pair of operators*, Proc. Roy. Soc. Edinburgh, **105A** (1987), 101-115.

[8] J. P. Dauer and P. Balasubramaniam, *Null Controllability of semilinear integrodifferential system in Banach space*, Appl. Math. Lett., **10** (1997), 117-123.

[9] J. C. Louis and D. Wexler, *On exact controllability in Hilbert spaces*, J. Diff. Eqns., **49** (1983), 258-269.

[10] N. I. Mahmudov, *Controllability of linear stochastic systems*, IEEE Trans. on Automatic Control, **46** (2001), 724-731.

[11] R. Rabah and J. Karrakchou, *Exact controllability and complete stabilizability for linear systems in Hilbert spaces*, Appl. Math. Lett., **10** (1997), 35-40.

[12] N. Sauer, *Linear evolution equations in two Banach spaces*, Proc. Roy. Soc. Edinburgh, **91A** (1982), 287-303.

[13] V. M. Wonham, *Random differential equations in control theory, Probabilistic Methods in Applied Mathematics*, Vol.**2**, Academic Press, New York, 1970.

[14] J. Zabczyk, *Controllability of Stochastic Linear Systems, Stochastic Differential Systems*, Lecture Notes in Control and Information Sciences, Springer-Verlag, New York, **43** (1982), 144-154.