LIBERTAS MATHEMATICA, vol XXXI (2011)

A Generalization of Blurly (1,2)-β-Irresolute Functions

Takashi NOIRI and Valeriu POPA

Abstract. In this paper we introduce the notion of weakly (1,2)-M-continuous functions as functions from a set satisfying some minimal conditions into a set satisfying some minimal conditions. We obtain some characterizations and several properties of such functions. This function leads to the formulation of a generalization of blurly (1,2)-β-irresolute functions defined in [4].

Keywords: m-structure, m-space, weakly M-continuous, (1,2)-semi-preopen, blurly (1,2)-β-irresolute.

Mathematics Subject Classification (2000): 54C08, 54E55.

1 Introduction

Semi-open sets, preopen sets, α-open sets and β-open sets play an important role in the researching of generalizations of continuity in topological spaces and bitopological spaces. By using these sets many authors introduced and studied various types of modifications of continuity in topological spaces and bitopological spaces. The notions of quasi-open sets or $\tau_1\tau_2$-open sets and quasi-continuity or $\tau_1\tau_2$-continuity are studied in [9], [15] and [29]. The notions of (1,2)-semi-open sets, (1,2)-preopen sets, and (1,2)-α-open sets and also (1,2)-semi-continuity, (1,2)-precontinuity, and (1,2)-α-continuity are introduced in [11]. The notions of (1,2)-semi-preopen sets and (1,2)-semi-precontinuity are introduced and studied in [12] and [27]. In 1961, Levine [13] introduced the concept of weakly continuous functions in topological spaces. By using the notions of semi-open sets, preopen sets, α-sets and β-open sets, several authors introduced and studied various forms of weak continuity in topological spaces and bitopological spaces. Quite recently, a new form of weak continuity is introduced and studied in [4].

In [24], [25] and [26], the present authors introduced and investigated the notions of minimal structures, m-spaces, M-continuity and weak M-continuity. By using the notions of minimal structures and m-spaces, the present authors [21], [22] introduced and investigated new forms of weak continuity in bitopological spaces. The purpose of this paper is to introduce the notions of weakly (1,2)-M-continuous functions which generalize the notions of blurly (1,2)-β-irresolute functions [4] and to transfer the study of weakly (1,2)-M-continuous functions between bitopological spaces to the study of weakly M-continuous functions between m-spaces.

2 Preliminaries

Let (X, τ) be a topological space and A a subset of X. The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively.

Definition 2.1. Let (X, τ) be a topological space. A subset A of X is said to be
(1) *semi-open* [14] if $A \subset \mathrm{Cl}(\mathrm{Int}(A))$,
(2) *preopen* [17] if $A \subset \mathrm{Int}(\mathrm{Cl}(A))$,
(3) α*-open* [20] if $A \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$,
(4) β*-open* [1] or *semi-preopen* [3] if $A \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$.

The family of all semi-open (resp. preopen, α-open, β-open) sets in (X, τ) is denoted by $\mathrm{SO}(X)$ (resp. $\mathrm{PO}(X)$, $\alpha(X)$, $\beta(X)$ or $\mathrm{SPO}(X)$).

Definition 2.2. The complement of a semi-open (resp. preopen, α-open, β-open) set is said to be *semi-closed* [6] (resp. *preclosed* [17], α*-closed* [18], β*-closed* [1] or *semi-preclosed* [3]).

Definition 2.3. The intersection of all semi-closed (resp. preclosed, α-closed, β-closed) sets of X containing A is called the *semi-closure* [6] (resp. *preclosure* [10], α*-closure* [18], β*-closure* [2] or *semi-preclosure* [3]) of A and is denoted by $\mathrm{sCl}(A)$ (resp. $\mathrm{pCl}(A)$, $\alpha\mathrm{Cl}(A)$, ${}_{\beta}\mathrm{Cl}(A)$ or $\mathrm{spCl}(A)$).

Definition 2.4. The union of all semi-open (resp. preopen, α-open, β-open) sets of X contained in A is called the *semi-interior* (resp. *preinterior*, α*-interior*, β*-interior* or *semi-preinterior*) of A and is denoted by $\mathrm{sInt}(A)$ (resp. $\mathrm{pInt}(A)$, $\alpha\mathrm{Int}(A)$, ${}_{\beta}\mathrm{Int}(A)$ or $\mathrm{spInt}(A)$).

Throughout the present paper, (X, τ) and (Y, σ) (or simply X and Y) denote topological spaces and (X, τ_1, τ_2) and (Y, σ_1, σ_2) denote bitopological spaces.

3 Minimal structures and M-continuity

Definition 3.1. A subfamily m_X of the power set $\mathcal{P}(X)$ of a nonempty set X is called a *minimal structure* (briefly m*-structure*) on X [24], [25] if $\emptyset \in m_X$ and $X \in m_X$.

By (X, m_X), we denote a nonempty subset X with a minimal structure m_X on X and call it an m-space. Each member of m_X is said to be m_X*-open* (briefly m-open) and the complement of an m_X-open set is said to be m_X*-closed* (briefly m-closed).

Remark 3.1. Let (X, τ) be a topological space. Then the families τ, $\mathrm{SO}(X)$, $\mathrm{PO}(X)$, $\alpha(X)$, $\beta(X)$, $\mathrm{SPO}(X)$ are all m-structures on X.

Definition 3.2. Let X be a nonempty set and m_X an m-structure on X. For a subset A of X, the m_X*-closure* of A and the m_X*-interior* of A are defined in [16] as follows:
(1) $\mathrm{mCl}(A) = \bigcap\{F : A \subset F, X \setminus F \in m_X\}$,
(2) $\mathrm{mInt}(A) = \bigcup\{U : U \subset A, U \in m_X\}$.

Remark 3.2. Let (X, τ) be a topological space and A a subset of X. If $m_X = \tau$ (resp. $\mathrm{SO}(X)$, $\mathrm{PO}(X)$, $\alpha(X)$, $\beta(X)$, $\mathrm{SPO}(X)$), then we have
(1) $\mathrm{mCl}(A) = \mathrm{Cl}(A)$ (resp. $\mathrm{sCl}(A)$, $\mathrm{pCl}(A)$, $\alpha\mathrm{Cl}(A)$, ${}_\beta\mathrm{Cl}(A)$, $\mathrm{spCl}(A)$),
(2) $\mathrm{mInt}(A) = \mathrm{Int}(A)$ (resp. $\mathrm{sInt}(A)$, $\mathrm{pInt}(A)$, $\alpha\mathrm{Int}(A)$, ${}_\beta\mathrm{Int}(A)$, $\mathrm{spInt}(A)$).

Lemma 3.1. (Maki [16]) *Let X be a nonempty set and m_X an m-structure on X. For subsets A and B of X, the following properties hold:*
(1) $\mathrm{mCl}(X \setminus A) = X \setminus \mathrm{mInt}(A)$ *and* $\mathrm{mInt}(X \setminus A) = X \setminus \mathrm{mCl}(A)$,
(2) *If* $(X \setminus A) \in m_X$, *then* $\mathrm{mCl}(A) = A$ *and if* $A \in m_X$, *then* $\mathrm{mInt}(A) = A$,
(3) $\mathrm{mCl}(\emptyset) = \emptyset$, $\mathrm{mCl}(X) = X$, $\mathrm{mInt}(\emptyset) = \emptyset$ *and* $\mathrm{mInt}(X) = X$,
(4) *If* $A \subset B$, *then* $\mathrm{mCl}(A) \subset \mathrm{mCl}(B)$ *and* $\mathrm{mInt}(A) \subset \mathrm{mInt}(B)$,
(5) $A \subset \mathrm{mCl}(A)$ *and* $\mathrm{mInt}(A) \subset A$,
(6) $\mathrm{mCl}(\mathrm{mCl}(A)) = \mathrm{mCl}(A)$ *and* $\mathrm{mInt}(\mathrm{mInt}(A)) = \mathrm{mInt}(A)$.

Lemma 3.2. (Popa and Noiri [24]) *Let (X, m_X) be an m-space and A a subset of X. Then $x \in \mathrm{mCl}(A)$ if and only if $U \cap A \neq \emptyset$ for every $U \in m_X$ containing x.*

Definition 3.3. An m-structure m_X on a nonempty set X is said to have *property $\mathcal{B}$* [16] if the union of any family of subsets belonging to m_X belongs to m_X.

Lemma 3.3. (Popa and Noiri [26]) *Let X be a nonempty set and m_X an m-structure on X satisfying property $\mathcal{B}$. For a subset A of X, the following properties hold:*
(1) $A \in m_X$ *if and only if* $\mathrm{mInt}(A) = A$,
(2) A *is* m_X*-closed if and only if* $\mathrm{mCl}(A) = A$,
(3) $\mathrm{mInt}(A) \in m_X$ *and* $\mathrm{mCl}(A)$ *is* m_X*-closed.*

Definition 3.4. A function $f : (X, m_X) \to (Y, m_Y)$ is said to be *weakly M-continuous* [26] (resp. *M-continuous* [24]) at $x \in X$ if for each $V \in m_Y$ containing $f(x)$, there exists $U \in m_X$ containing x such that $f(U) \subset \mathrm{mCl}(V)$ (resp. $f(U) \subset V$). A function $f : (X, m_X) \to (Y, m_Y)$ is said to be weakly *M-continuous* (resp. *M-continuous*) if it has the property at each point $x \in X$.

Theorem 3.1. *For a function $f : (X, m_X) \to (Y, m_Y)$, where m_Y has property $\mathcal{B}$, the following properties are equivalent:*
(1) *f is weakly M-continuous at $x \in X$;*
(2) *for every $V \in m_Y$ with $x \in f^{-1}(V), x \in \mathrm{mInt}(f^{-1}(\mathrm{mCl}(V)))$;*
(3) *for every m_Y-closed set F of Y with $x \in \mathrm{mCl}(f^{-1}(\mathrm{mInt}(F))), x \in f^{-1}(F)$;*
(4) *for every subset B of Y with $x \in \mathrm{mCl}(f^{-1}(\mathrm{mInt}(\mathrm{mCl}(B)))), x \in f^{-1}(\mathrm{mCl}(B))$;*
(5) *for every subset B of Y with $x \in f^{-1}(\mathrm{mInt}(B)), x \in \mathrm{mInt}(f^{-1}(\mathrm{mCl}(\mathrm{mInt}(B))))$;*
(6) *for every m_Y-open set V with $x \in \mathrm{mCl}(f^{-1}(V)), x \in f^{-1}(\mathrm{mCl}(V))$.*

Proof. (1) $\Rightarrow$ (2): Let f be weakly M-continuous at x and $V \in m_Y$ such that $x \in f^{-1}(V)$. Then, $f(x) \in V$ and there exists $U \in m_X$ containing x such that $f(U) \subset \mathrm{mCl}(V)$. Then we have $x \in U \subset f^{-1}(\mathrm{mCl}(V))$. This implies that $x \in \mathrm{mInt}(f^{-1}(\mathrm{mCl}(V)))$.

(2) $\Rightarrow$ (3): Let F be any m_Y-closed set of Y. Suppose that $x \notin f^{-1}(F)$. Then $Y \setminus F \in m_Y$ and $x \in X \setminus f^{-1}(F) = f^{-1}(Y \setminus F)$. By (2) and Lemma 3.1, we have

$$\begin{aligned} x &\in \mathrm{mInt}(f^{-1}(\mathrm{mCl}(Y \setminus F))) = \mathrm{mInt}(f^{-1}(Y \setminus \mathrm{mInt}(F))) \\ &= \mathrm{mInt}(X \setminus f^{-1}(\mathrm{mInt}(F))) = X \setminus \mathrm{mCl}(f^{-1}(\mathrm{mInt}(F))). \end{aligned}$$

Therefore, we obtain $x \notin \mathrm{mCl}(f^{-1}(\mathrm{mInt}(F)))$.

(3) $\Rightarrow$ (4): Let B be any subset of Y. Since m_Y has property $\mathcal{B}$, $\mathrm{mCl}(B)$ is m_Y-closed and for any $x \in \mathrm{mCl}(f^{-1}(\mathrm{mInt}(\mathrm{mCl}(B)))), x \in f^{-1}(\mathrm{mCl}(B))$.

(4) $\Rightarrow$ (5): Let B be any subset of Y and $x \in f^{-1}(\mathrm{mInt}(B))$. Then we have $x \in f^{-1}(\mathrm{mInt}(B)) = X \setminus f^{-1}(\mathrm{mCl}(Y \setminus B))$. Then $x \notin f^{-1}(\mathrm{mCl}(Y \setminus B))$. Then, by (4) $x \in X \setminus \mathrm{mCl}(f^{-1}(\mathrm{mInt}(\mathrm{mCl}(Y \setminus B))) = \mathrm{mInt}(f^{-1}(\mathrm{mCl}(\mathrm{Int}(B)))$.

(5) $\Rightarrow$ (6): Let V be any m_Y-open set of Y. Suppose that $x \notin f^{-1}(\mathrm{mCl}(V))$. Then $f(x) \notin \mathrm{mCl}(V)$ and by Lemma 3.2 there exists $W \in m_Y$ containing $f(x)$ such that $W \cap V = \emptyset$; hence $\mathrm{mCl}(W) \cap V = \emptyset$. By (5), we have $x \in \mathrm{mInt}(f^{-1}(\mathrm{mCl}(W)))$ and hence there exists $U \in m_X$ such that $x \in U \subset f^{-1}(\mathrm{mCl}(W))$. Since $\mathrm{mCl}(W) \cap V = \emptyset$, $U \cap f^{-1}(V) = \emptyset$ and by Lemma 3.2 $x \notin \mathrm{mCl}(f^{-1}(V))$. Therefore, for each $x \in \mathrm{mCl}(f^{-1}(V)), x \in f^{-1}(\mathrm{mCl}(V))$.

(6) $\Rightarrow$ (1): Let V be any m_Y-open set containing $f(x)$. Then we have $x \in f^{-1}(V) \subset f^{-1}(\mathrm{mInt}(\mathrm{mCl}(V))) = X \setminus f^{-1}(\mathrm{mCl}(Y \setminus \mathrm{mCl}(V)))$. By (6), $x \notin \mathrm{mCl}(f^{-1}(Y \setminus \mathrm{mCl}(V)))$ and $x \in \mathrm{mInt}(f^{-1}(\mathrm{mCl}(V)))$. Therefore, there exists $U \in m_X$ such that $x \in U \subset f^{-1}(\mathrm{mCl}(V))$; hence $f(U) \subset \mathrm{mCl}(V)$. This shows that f is weakly M-continuous at $x \in X$.

Definition 3.5. Let S be a subset of an m-space (X, m_X). A point $x \in X$ is called an *m_θ-adherent* point of S if $\mathrm{mCl}(U) \cap S \neq \emptyset$ for every m_X-open set U containing x.

The set of all m_θ-adherent points of S is called the *m_θ-closure* of S and is denoted by $\mathrm{mCl}_\theta(S)$. If $S = \mathrm{mCl}_\theta(S)$, then S is said to be *m_θ-closed*. The complement of an m_θ-closed set is said to be *m_θ-open*.

Remark 3.3. Let S be a subset of a topological space (X, τ) and $m_X = \tau$ (resp. $\mathrm{SO}(X)$, $\mathrm{PO}(X)$), then $\mathrm{mCl}_\theta(S) = \mathrm{Cl}_\theta(S)$ [30] (resp. $\mathrm{sCl}_\theta(S)$ [7], $\mathrm{pCl}_\theta(S)$ [23]).

Lemma 3.4. (Popa and Noiri [26]) *Let A be a subset of an m-space (X, m_X). Then the following properties hold:*

(1) *If A is m_X-open in (X, m_X), then* $\mathrm{mCl}(A) = \mathrm{mCl}_\theta(A)$,

(2) *If m_X satisfies property $\mathcal{B}$, then $\mathrm{mCl}_\theta(A)$ is m_X-closed for every subset A of X.*

Theorem 3.2. (Popa and Noiri [26], [21]) *For a function $f : (X, m_X) \to (Y, m_Y)$, the following properties are equivalent:*

(1) *f is weakly M-continuous;*

(2) $f^{-1}(V) \subset \mathrm{mInt}(f^{-1}(\mathrm{mCl}(V)))$ *for every $V \in m_Y$;*

(3) $f(\mathrm{mCl}(A)) \subset \mathrm{mCl}_\theta(f(A))$ *for every subset A of X;*

(4) $\mathrm{mCl}(f^{-1}(B)) \subset f^{-1}(\mathrm{mCl}_\theta(B))$ *for every subset B of Y;*

(5) $\mathrm{mCl}(f^{-1}(\mathrm{mInt}(\mathrm{mCl}(B)))) \subset f^{-1}(\mathrm{mCl}(B))$ *for every subset B of Y;*

(6) $\mathrm{mCl}(f^{-1}(\mathrm{mInt}(F))) \subset f^{-1}(F)$ *for every m_Y-closed set F of Y.*

Theorem 3.3. (Popa and Noiri [26]) *Let (Y, m_Y) be an m-space and m_Y satisfy $\mathcal{B}$. Then, a function $f : (X, m_X) \to (Y, m_Y)$ is weakly M-continuous if and only if* $\mathrm{mCl}(f^{-1}(V)) \subset f^{-1}(\mathrm{mCl}(V))$ *for every m_Y-open set V of Y.*

For a function $f : (X, m_X) \to (Y, m_Y)$, we define $D_{WMC}(f)$ as follows:

$$D_{WMC}(f) = \{x \in X : f \text{ is not weakly } M\text{-continuous at } x\}.$$

Theorem 3.4. *For a function $f : (X, m_X) \to (Y, m_Y)$, where m_Y has property $\mathcal{B}$, the following equalities hold:*

$$\begin{aligned} D_{WMC}(f) &= \bigcup_{G \in m_Y} \{f^{-1}(G) \setminus \mathrm{mInt}(f^{-1}(\mathrm{mCl}(G)))\} \\ &= \bigcup_{H \in \mathcal{F}} \{\mathrm{mCl}(f^{-1}(\mathrm{mInt}(H))) \setminus f^{-1}(H)\} \\ &= \bigcup_{B \in \mathcal{P}(Y)} \{\mathrm{mCl}(f^{-1}(\mathrm{mInt}(\mathrm{mCl}(B)))) \setminus f^{-1}(\mathrm{mCl}(B))\} \\ &= \bigcup_{B \in \mathcal{P}(Y)} \{f^{-1}(\mathrm{mInt}(B)) \setminus \mathrm{mInt}(f^{-1}(\mathrm{mCl}(\mathrm{mInt}(B))))\} \\ &= \bigcup_{G \in m_Y} \{\mathrm{mCl}(f^{-1}(G)) \setminus f^{-1}(\mathrm{mCl}(G))\}, \end{aligned}$$

where $\mathcal{F}$ is the family of m_Y-closed sets of (Y, m_Y).

Proof. We shall show only the first equality since the proofs of other are similar to the first. Let $x \in D_{WMC}(f)$. By Theorem 3.1, there exists $V \in m_Y$ such that $x \in f^{-1}(V)$ and $x \notin \mathrm{mInt}(f^{-1}(\mathrm{mCl}(V)))$. Therefore, we have

$$x \in f^{-1}(V) \setminus \mathrm{mInt}(f^{-1}(\mathrm{mCl}(V))) \subset \bigcup_{G \in m_Y} \{f^{-1}(G) \setminus \mathrm{mInt}(f^{-1}(\mathrm{mCl}(G)))\}.$$

Conversely, let $x \in \bigcup_{G \in m_Y} \{f^{-1}(G) - \mathrm{mInt}(f^{-1}(\mathrm{mCl}(G)))\}$. Then, there exists $V \in m_Y$ such that $x \in f^{-1}(V) \setminus \mathrm{mInt}(f^{-1}(\mathrm{mCl}(V)))$. By Theorem 3.1, we obtain $x \in D_{WMC}(f)$.

Definition 3.6. Let (X, m_X) be an m-space and A a subset of X. The m-frontier of A, $\mathrm{mFr}(A)$, is defined as follows: $\mathrm{mFr}(A) = \mathrm{mCl}(A) \cap \mathrm{mCl}(X \setminus A) = \mathrm{mCl}(A) \setminus \mathrm{mInt}(A)$.

Theorem 3.5. (Popa and Noiri [26]) *The set of all points $x \in X$ at which a function $f : (X, m_X) \to (Y, m_Y)$ is not weakly M-continuous is identical with the union of the m-frontiers of the inverse images of the m_Y-closures of m_Y-open sets containing f(x).*

4 Minimal structures and bitopological spaces

Definition 4.1. A subset A of a bitopological space (X, τ_1, τ_2) is said to be *quasi-open* [9], [29] or *$\tau_1\tau_2$-open* [11] if $A = B \cup C$, where $B \in \tau_1$ and $C \in \tau_2$.

The complement of a $\tau_1\tau_2$-open set is said to be *$\tau_1\tau_2$-closed.* The intersection of all $\tau_1\tau_2$-closed sets containing a subset A of X is called the *$\tau_1\tau_2$-closure* and is denoted by $\tau_1\tau_2\mathrm{Cl}(A)$. The union of all $\tau_1\tau_2$-open sets of X contained in A is called the *$\tau_1\tau_2$-interior* of A and is denoted by $\tau_1\tau_2\mathrm{Int}(A)$. The family of all $\tau_1\tau_2$-open sets of (X, τ_1, τ_2) is denoted by (1,2)O(X).

Definition 4.2. A subset A of a bitopological space (X, τ_1, τ_2) is said to be
(1) *(1,2)-semi-open* [11] if $A \subset \tau_1\tau_2\mathrm{Cl}(\tau_1\mathrm{Int}(A))$,
(2) *(1,2)-preopen* [11] if $A \subset \tau_1\mathrm{Int}(\tau_1\tau_2\mathrm{Cl}(A))$,
(3) *(1,2)-α-open* [11] if $A \subset \tau_1\mathrm{Int}(\tau_1\tau_2\mathrm{Cl}(\tau_1\mathrm{Int}(A)))$,
(4) *(1,2)-semi-preopen* [12], [27] if $A \subset \tau_1\tau_2\mathrm{Cl}(\tau_1\mathrm{Int}(\tau_1\tau_2\mathrm{Cl}(A)))$.

The complement of (1,2)-semi-open (resp. (1,2)-preopen, (1,2)-α-open, (1,2)-semi-preopen) set of X is said to be *(1,2)-semi-closed* (resp. *(1,2)-preclosed*, *(1,2)-α-closed*, *(1,2)-semi-preclosed*). The intersection of all (1,2)-semi-closed (resp. (1,2)-preclosed, (1,2)-α-closed, (1,2)-semi-preclosed) sets containing A is called the *(1,2)-semi-closure* (resp. *(1,2)-preclosure*, *(1,2)-α-closure*, *(1,2)-semi-preclosure*) of A and is denoted by (1,2)sCl(A) (resp. (1,2)pCl(A), (1,2)αCl(A), (1,2)spCl(A)). The union of (1,2)-semi-open (resp. (1,2)-preopen, (1,2)-α-open, (1,2)-semi-preopen) sets of X contained in A is called the *(1,2)-semi-interior* (resp. *(1,2)-preinterior*, *(1,2)-α-interior*, *(1,2)-semi-preinterior*) of A and is denoted by (1,2)sInt(A) (resp. (1,2)pInt(A), (1,2)αInt(A), (1,2)spInt(A)).

The collection of all (1,2)-semi-open (resp. (1,2)-preopen, (1,2)-α-open, (1,2)-semi-preopen) sets of X is denoted by (1,2)SO(X) (resp. (1,2)PO(X), (1,2)αO(X), (1,2)SPO(X)).

Remark 4.1. Let (X, τ_1, τ_2) be a bitopological space and A a subset of X.

(1) $\tau_1\tau_2$O(X), (1,2)SO(X), (1,2)PO(X), (1,2)αO(X) and (1,2)SPO(X) are all m-structures on X having property $\mathcal{B}$.

(2) By (1,2)$\mathcal{M}(X)$, we denote the collection of all m-structures determined by the topologies τ_1 and τ_2 on X and call each member of (1,2)$\mathcal{M}(X)$ a (1,2)M-structure on X. Let (1,2)M(X) $\in (1,2)\mathcal{M}(X)$. If (1,2)M(X) $= \tau_1\tau_2$O(X) (resp. (1,2)SO(X), (1,2)PO(X), (1,2)αO(X), (1,2)SPO(X)), then we have

i) (1,2)mCl(A) $= \tau_1\tau_2$Cl(A) (resp. (1,2)sCl(A), (1,2)pCl(A), (1,2)αCl(A), (1,2)spCl(A)),

ii) (1,2)mInt(A) $= \tau_1\tau_2$Int(A) (resp. (1,2)sInt(A), (1,2)pInt(A), (1,2)αInt(A), (1,2)spInt(A)).

(3) A subset A is $\tau_1\tau_2$-closed (resp. (1,2)-semi-closed, (1,2)-preclosed, (1,2)-α-closed, (1,2)-semi-preclosed) if and only if $A = \tau_1\tau_2$Cl(A) (resp. $A =$ (1,2)sCl(A), $A =$ (1,2)pCl(A), $A =$ (1,2)αCl(A), $A =$ (1,2)spCl(A)).

(4) A is $\tau_1\tau_2$-open (resp. (1,2)-semi-open, (1,2)-preopen, (1,2)-α-open, (1,2)-semi-preopen) if and only if $A = \tau_1\tau_2$Int(A) (resp. $A =$ (1,2)sInt(A), $A =$ (1,2)pInt(A), $A =$ (1,2)αInt(A), $A =$ (1,2)spInt(A)).

(5) By Lemma 3.2, we obtain the results established in Lemma 8(iii) of [4].

(6) By Lemma 3.1, we obtain the relations between (1,2)mCl(A) and (1,2)mInt(A).

(7) Let (1,2)M(X) $\in (1,2)\mathcal{M}(X)$ and A be a subset of X. By (1,2)mCl$_\theta$(A), we denote the m_θ-closure of A in (1,2)M(X).

5 Weakly (1,2)-M-continuous functions

Definition 5.1. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be *blurly (1,2)-β-irresolute* [4] if for each point $x \in X$ and each $V \in (1,2)\mathrm{SPO}(Y)$ containing $f(x)$, there exists $U \in (1,2)\mathrm{SPO}(X)$ containing x such that $f(U) \subset (1,2)\mathrm{spCl}(V)$.

Hence, it turns out that $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is blurly (1,2)-β-irresolute if and only if $f : (X, (1,2)\mathrm{SPO}(X)) \to (Y, (1,2)\mathrm{SPO}(Y))$ is weakly M-continuous.

Definition 5.2. Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be bitopological spaces and (1,2)M(X) (resp. (1,2)M(Y)) a (1,2)M-structure on X (resp. Y). A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$

is said to be *(1,2)-M-continuous* or *(1,2)-M-irresolute* (resp. *weakly (1,2)-M-continuous* or *blurly (1,2)-M-irresolute* at a point $x \in X$ (resp. on X) if $f : (X, (1,2)\mathrm{M}(X)) \to (Y, (1,2)\mathrm{M}(Y))$ is M-continuous (resp. weakly M-continuous) at $x \in X$ (resp. on X).

Remark 5.1. (1) If (1,2)M(X) = (1,2)SPO(X) and (1,2)M(Y) = (1,2)SPO(Y), then we obtain the definitions of (1,2)-β-irresolute functions [19] and blurly (1,2)-β-irresolute functions [4].

(2) If (1,2)M(X) $= (1,2)\mathrm{O}(X)$ (resp. (1,2)SO(X), (1,2)PO(X), (1,2)αO(X), (1,2)SPO(X)), $(1,2)\mathrm{O}(Y)$ (resp. (1,2)SO(Y), (1,2)PO(Y), (1,2)αO(Y), (1,2)SPO(Y)) and $f : (X, (1,2)M(X)) \to (Y, (1,2)M(Y))$ is (1,2)-M-continuous or weakly (1,2)-M-continuous, then we obtain the definitions of a (1,2)-continuous (resp. (1,2)-semi-continuous, (1,2)-precontinuous, (1,2)-α-continuous, (1,2)-semi-precontinuous) function or a weakly (1,2)-continuous (resp. weakly (1,2)-semi-continuous, weakly (1,2)-precontinuous, weakly (1,2)-α-continuous, weakly (1,2)-semi-precontinuous) function.

(3) (1,2)-M-continuity (or weak (1,2)-M-continuity) is a unified form of the functions stated in (1) and (2).

The following three theorems follow from Theorems 3.1, 3.2 and 3.3.

Theorem 5.1. *Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be bitopological spaces and (1,2)M(X) (resp. (1,2)M(Y)) a (1,2)M-structure on X (resp. Y), where (1,2)M(Y) has property $\mathcal{B}$. For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *f is weakly (1,2)-M-continuous at $x \in X$;*

(2) *for every (1,2)M-open set V of Y with $x \in f^{-1}(V), x \in (1,2)\mathrm{mInt}(f^{-1}((1,2)\mathrm{mCl}(V)))$;*

(3) *for every (1,2)M-closed set F of Y with $x \in (1,2)\mathrm{mCl}(f^{-1}((1,2)\mathrm{mInt}(F))), x \in f^{-1}(F)$;*

(4) *for every subset B of Y with $x \in (1,2)\mathrm{mCl}(f^{-1}((1,2)\mathrm{mInt}((1,2)\mathrm{mCl}(B)))), x \in f^{-1}((1,2)\mathrm{mCl}(B))$;*

(5) *for every subset B of Y with $x \in f^{-1}((1,2)\mathrm{mInt}(B))$, $x \in (1,2)\mathrm{mInt}(f^{-1}((1,2)\mathrm{mCl}((1,2)\mathrm{mInt}(B))))$;*

(6) *for every (1,2)M-open set V with $x \in (1,2)\mathrm{mCl}(f^{-1}(V)), x \in f^{-1}((1,2)\mathrm{mCl}(V))$.*

Theorem 5.2. *Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be bitopological spaces and (1,2)M(X) (resp. (1,2)M(Y)) a (1,2)M-structure on X (resp. Y). For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *f is weakly (1,2)-M-continuous;*

(2) $f^{-1}(V) \subset (1,2)\mathrm{mInt}(f^{-1}((1,2)\mathrm{mCl}(V)))$ *for every (1,2)M-open set V of Y;*

(3) $f((1,2)\mathrm{mCl}(A)) \subset (1,2)\mathrm{mCl}_\theta(f(A))$ *for every subset A of X;*

(4) $(1,2)\mathrm{mCl}(f^{-1}(B)) \subset f^{-1}((1,2)\mathrm{mCl}_\theta(B))$ *for every subset B of Y;*

(5) $(1,2)\mathrm{mCl}(f^{-1}((1,2)\mathrm{mInt}((1,2)\mathrm{mCl}(B)))) \subset f^{-1}((1,2)\mathrm{mCl}(B))$ *for every subset B of Y;*

(6) $(1,2)\mathrm{mCl}(f^{-1}((1,2)\mathrm{mInt}(F))) \subset f^{-1}(F)$ *for every (1,2)M-closed set F of Y.*

Theorem 5.3. *Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be bitopological spaces and (1,2)M(X) (resp. (1,2)M(Y)) a (1,2)M-structure on X (resp. Y), where (1,2)M(Y) has property $\mathcal{B}$. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is weakly (1,2)-M-continuous if and only if $(1,2)\mathrm{mCl}(f^{-1}(V)) \subset f^{-1}((1,2)\mathrm{mCl}(V))$ for every (1,2)M-open set V of Y.*

Remark 5.2. (1) If (1,2)M(X) = (1,2)SPO(X) and (1,2)M(Y) = (1,2)SPO(Y), then by Theorems 5.2 and 5.3 we obtain the results established in Theorem 29 of [4] and Theorem 31 (i), (ii), (iii) of [4].

(2) The results of Theorem 34 of [4] are not correct (see Definition 30 and Theorem 3.5 of [5]).

Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be bitopological spaces and (1,2)M(X) (resp. (1,2)M(Y)) a (1,2)M-structure on X (resp. Y). For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, we define $D_{(1,2)WMC}(f)$ as follows:

$$D_{(1,2)WMC}(f) = \{x \in X : f \text{ is not weakly (1,2)-}M\text{-continuous at } x\}.$$

By Theorem 3.4, we obtain the following theorem.

Theorem 5.4. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, where $(1,2)\mathrm{M}(Y)$ has property $\mathcal{B}$, the following equalities hold:*

$$\begin{aligned}
D_{(1,2)WMC}(f) &= \textstyle\bigcup_{G \in (1,2)\mathrm{M}(Y)} \{f^{-1}(G) \setminus (1,2)\mathrm{mInt}(f^{-1}((1,2)\mathrm{mCl}(G)))\} \\
&= \textstyle\bigcup_{H \in \mathcal{F}} \{(1,2)\mathrm{mCl}(f^{-1}((1,2)\mathrm{mInt}(H))) \setminus f^{-1}(H)\} \\
&= \textstyle\bigcup_{B \in \mathcal{P}(Y)} \{(1,2)\mathrm{mCl}(f^{-1}((1,2)\mathrm{mInt}((1,2)\mathrm{mCl}(B)))) \setminus f^{-1}((1,2)\mathrm{mCl}(B))\} \\
&= \textstyle\bigcup_{B \in \mathcal{P}(Y)} \{f^{-1}((1,2)\mathrm{mInt}(B)) \setminus (1,2)\mathrm{mInt}(f^{-1}((1,2)\mathrm{mCl}((1,2)\mathrm{mInt}(B)))))\} \\
&= \textstyle\bigcup_{G \in (1,2)\mathrm{M}(Y)} \{(1,2)\mathrm{mCl}(f^{-1}(G)) \setminus f^{-1}((1,2)\mathrm{mCl}(G))\},
\end{aligned}$$

where $\mathcal{F}$ is the family of (1,2)M-closed sets of Y.

Definition 5.3. Let (X, τ_1, τ_2) be a bitopological space, A a subset of X and (1,2)M(X) a (1,2)M-structure on X. Then the *(1,2)M-frontier* of A is defined as follows: (1,2)MFr(A) = (1,2)mCl(A) $\cap$ $(1,2)\mathrm{mCl}(X \setminus A)$ = (1,2)mCl(A) $\setminus$ $(1,2)\mathrm{mInt}(A)$.

By Definition 5.3 and Theorem 3.5, we obtain the following theorem.

Theorem 5.5. *Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a function. Then $D_{(1,2)WMC}(f)$ is identical with the union of the (1,2)M-frontiers of the inverse images of the (1,2)M-closures of (1,2)M-open sets of Y containing f(x).*

6 Weak (1,2)-M-continuity and (1,2)-M-continuity

In this section, we investigate the relationships between (1,2)-M-continuous functions and weakly (1,2)-M-continuous functions.

Definition 6.1. A function $f : (X, m_X) \to (Y, m_Y)$ is said to satisfy the *m-interiority condition* [26] if $\mathrm{mInt}(f^{-1}(\mathrm{mCl}(V))) \subset f^{-1}(V)$ for each m_Y-open set V of Y.

Lemma 6.1. (Popa and Noiri [26]) *If a function $f : (X, m_X) \to (Y, m_Y)$ is weakly M-continuous and satisfies m-interiority condition, then f is M-continuous.*

Definition 6.2. Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be bitopological spaces and (1,2)M(X) (resp. (1,2)M(Y)) a (1,2)M-structure on X (resp. Y). A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to satisfy *(1,2)M-interiority condition* if $f : (X, (1,2)\mathrm{M}(X)) \to (Y, (1,2)\mathrm{M}(Y))$ satisfies m-interiority condition.

Therefore, a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ satisfies (1,2)M-interiority condition if (1,2)mInt($f^{-1}((1,2)\mathrm{mCl}(V))) \subset f^{-1}(V)$ for every (1,2)M-open set V of Y.

Theorem 6.1. *Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be bitopological spaces and (1,2)M(X) (resp. (1,2)M(Y)) a (1,2)M-structure on X (resp. Y). If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is weakly (1,2)-M-continuous and satisfies (1,2)M-interiority condition, then f is (1,2)M-continuous.*

Proof. The proof is an immediate consequence of Lemma 6.1.

Definition 6.3. An m-space (X, m_X) is said to be *m-regular* [26] if for each m_X-closed set F and each $x \notin F$, there exist disjoint m_X-open sets U and V such that $x \in U$ and $F \subset V$.

Remark 6.1. Let (X, τ) be a topological space and $m_X = \tau$ (resp. SO(X), PO(X)). Then m-regularity coincides with regularity (resp. semi-regularity [8], pre-regularity [23]).

Lemma 6.2. (Popa and Noiri [26]) *Let (Y, m_Y) be an m-regular m-space and satisfy property $\mathcal{B}$. Then, for a function $f : (X, m_X) \to (Y, m_Y)$, the following properties are equivalent:*
(1) *f is M-continuous;*
(2) $f^{-1}(\mathrm{mCl}_\theta(B)) = \mathrm{mCl}(f^{-1}(\mathrm{mCl}_\theta(B)))$ *for every subset B of Y;*
(3) *f is weakly M-continuous;*
(4) $f^{-1}(F) = \mathrm{mCl}(f^{-1}(F))$ *for every m_θ-closed set F of Y;*
(5) $f^{-1}(V) = \mathrm{mInt}(f^{-1}(V))$ *for every m_θ-open set V of Y.*

Definition 6.4. Let (X, τ_1, τ_2) be a bitopological space and (1,2)M(X) a (1,2)M-structure on X. Then (X, τ_1, τ_2) is said to be *(1,2)M-regular* if $(X, (1,2)\mathrm{M}(X))$ is m-regular, equivalently if for each (1,2)M-closed set F of (X, τ_1, τ_2) and each $x \notin F$, there exist disjoint (1,2)M-open sets U and V such that $x \in U$ and $F \subset V$.

Remark 6.2. If (X, τ_1, τ_2) is (1,2)M-regular and (1,2)M(X) = (1,2)SPO(X), then (X, τ_1, τ_2) is (1,2)-semi-preregular [4].

Theorem 6.2. *Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be bitopological spaces and (1,2)M(X) (resp. (1,2)M(Y)) a (1,2)M-structure on X (resp. Y), where (1,2)M(Y) satisfies property $\mathcal{B}$ and (Y, σ_1, σ_2) is (1,2)M-regular. Then, for a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*
(1) *f is (1,2)-M-continuous;*
(2) $f^{-1}((1,2)\mathrm{mCl}_\theta(B)) = (1,2)\mathrm{mCl}(f^{-1}((1,2)\mathrm{mCl}_\theta(B)))$ *for every subset B of Y;*
(3) *f is weakly (1,2)-M-continuous;*
(4) $f^{-1}(F) = (1,2)\mathrm{mCl}(f^{-1}(F))$ *for every $(1,2)m_\theta$-closed set F of Y;*
(5) $f^{-1}(V) = (1,2)\mathrm{mInt}(f^{-1}(V))$ *for every $(1,2)m_\theta$-open set V of Y.*

Proof. The proof is an immediate consequence of Lemma 6.2.

Remark 6.3. If (1,2)M(X) = (1,2)SPO(X) and (1,2)M(Y) = (1,2)SPO(Y), then by Theorem 5.2 we obtain the result established in Theorem 37 of [4].

7 Properties of weakly (1,2)-M-continuous functions

We can obtain several properties of weakly (1,2)-M-continuous functions by using those of weakly M-continuous functions established in [26].

Definition 7.1. An m-space (X, m_X) is said to be m-T_2 [24] (resp. *m-Urysohn* [26]) if for each distinct points $x, y \in X$, there exist $U, V \in m_X$ containing x and y, respectively, such that $U \cap V = \emptyset$ (resp. $\mathrm{mCl}(U) \cap \mathrm{mCl}(V) = \emptyset$).

Definition 7.2. Let (X, τ_1, τ_2) be a bitopological space and (1,2)M(X) a (1,2)M-structure on X. Then (X, τ_1, τ_2) is said to be *(1,2)m-T_2* (resp. *(1,2)m-Urysohn*) if $(X, (1,2)\mathrm{M}(X))$ is m-T_2 (resp. m-Urysohn).

Remark 7.1. If (X, τ_1, τ_2) is (1,2)m-T_2 and (1,2)M(X) = (1,2)SPO(X), then (X, τ_1, τ_2) is (1,2)-β-T_2 [4].

Lemma 7.1. (Popa and Noiri [26]) *If $f : (X, m_X) \to (Y, m_Y)$ is a weakly M-continuous injection and (Y, m_Y) is m-Urysohn, then (X, m_X) is m-T_2.*

Theorem 7.1. *Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be bitopological spaces and (1,2)M(X) (resp. (1,2)M(Y)) a (1,2)M-structure on X (resp. Y). If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a weakly (1,2)-M-continuous injection and (Y, σ_1, σ_2) is (1,2)m-Urysohn, then (X, τ_1, τ_2) is (1,2)m-T_2.*

Proof. The proof is an immediate consequence of Lemma 7.1.

Definition 7.3. A function $f : (X, m_X) \to (Y, m_Y)$ is said to have a *strongly M-closed graph* [26] if for each $(x, y) \in (X \times Y) - \mathrm{G}(f)$, there exist an m_X-open set U containing x and an m_Y-open set V of Y containing y such that $[U \times \mathrm{mCl}(V)] \cap \mathrm{G}(f) = \emptyset$.

Definition 7.4. Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be bitopological spaces and (1,2)M(X) (resp. (1,2)M(Y)) a (1,2)M-structure on X (resp. Y). A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to have *strongly (1,2)M-closed graph* if $f : (X, (1,2)\mathrm{M}(X)) \to (Y, (1,2)\mathrm{M}(Y))$ has a strongly m-closed graph.

Lemma 7.2. (Popa and Noiri [26]) *Let $f : (X, m_X) \to (Y, m_Y)$ be a weakly M-continuous function. Then the following properties hold:*

(1) *If (Y, m_Y) is m-Urysohn, then* G(f) *is strongly M-closed,*
(2) *If f is injective and G(f) is strongly M-closed, then (X, m_X) is m-T_2.*

Theorem 7.2. *Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be bitopological spaces and (1,2)M(X) (resp. (1,2)M(Y)) a (1,2)M-structure on X (resp. Y). For a weakly (1,2)-M-continuous function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties hold:*

(1) *If (Y, σ_1, σ_2) is (1,2)m-Urysohn, then* G*(f) is strongly (1,2)M-closed,*
(2) *If f is injective and G(f) is strongly M-closed, then (X, τ_1, τ_2) is (1,2)m-T_2*

Proof. The proof is an immediate consequence of Lemma 7.2.

Definition 7.5. An m-spacce (X, m_X) is said to be *m-connected* [24] if X cannot be written as the union of two nonempty disjoint m_X-open sets.

Definition 7.6. Let (X,τ_1,τ_2) be a bitopological space and (1,2)M(X) a (1,2)M-structure on X. Then (X,τ_1,τ_2) is said to be *(1,2)M-connected* if $(X,(1,2)\mathrm{M}(X))$ is m-connected.

Lemma 7.3. (Popa and Noiri [26]) *If $f:(X,m_X)\to(Y,m_Y)$ is a weakly M-continuous surjection, where m_X satisfies $\mathcal{B}$ and (X,m_X) is m-connected, then (Y,m_Y) is m-connected.*

Theorem 7.3. *Let (X,τ_1,τ_2) and (Y,σ_1,σ_2) be bitopological spaces and (1,2)M(X) (resp. (1,2)M(Y)) a (1,2)M-structure on X (resp. Y), where (1,2)M(X) satisfies property $\mathcal{B}$. If $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is a weakly (1,2)-M-continuous surjection and (X,τ_1,τ_2) is (1,2)M-connected, then (Y,σ_1,σ_2) is (1,2)M-connected.*

Proof. The proof is an immediate consequence of Lemma 7.3.

Definition 7.7. A subset K of an m-space (X,m_X) is said to be *m-closed* [24] (resp. *m-compact* [24]) relative to (X,m_X) if for any cover $\{V_\alpha:\alpha\in\Delta\}$ of K by m_X-open sets of X, there exists a finite subset Δ_0 of Δ such that $K\subset\bigcup\{\mathrm{mCl}(V_\alpha):\alpha\in\Delta_0\}$ (resp. $K\subset\bigcup\{V_\alpha:\alpha\in\Delta_0\}$). If X is m-closed (resp. m-compact) relative to (X,m_X), then (X,m_X) is said to be *m-closed* (resp. *m-compact*).

Definition 7.8. Let (X,τ_1,τ_2) be a bitopological space and (1,2)M(X) a (1,2) M-structure on X. Then a subset K of (X,τ_1,τ_2) is said to be *(1,2)M-closed* (resp. *(1,2)M-compact*) relative to (X,τ_1,τ_2) if K is m-closed (resp. m-compact) relative to $(X,(1,2)\mathrm{M}(X))$.

Lemma 7.4. (Popa and Noiri [26]) *Let $f:(X,m_X)\to(Y,m_Y)$ be a weakly M-continuous function. Then the following properties hold:*

(1) *If K is m-compact relative to (X,m_X), then $f(K)$ is m-closed relative to (Y,m_Y),*

(2) *If f is a surjection and (X,m_X) is m-compact, then (Y,m_Y) is m-closed.*

Theorem 7.4. *Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a weakly (1,2)-M-continuous function and (1,2)M(X) (resp. (1,2)M(Y)) a (1,2)M-structure on X (resp. Y). Then the following properties hold:*

(1) *If K is (1,2)M-compact relative to (X,τ_1,τ_2), then $f(K)$ is (1,2)M-closed relative to (Y,σ_1,σ_2),*

(2) *If f is a surjection and (X,τ_1,τ_2) is (1,2)M-compact, then (Y,σ_1,σ_2) is (1,2)M-closed.*

Proof. The proof is an immediate consequence of Lemma 7.4.

References

[1] M. E. Abd El-Monsef, S. N. El-Deeb and R. A. Mahmoud, *β-open sets and β-continuous mappings*, Bull. Fac. Sci. Assiut Univ. **12** (1983), 77–90.

[2] M. E. Abd El-Monsef, R. A. Mahmoud and E. R. Lashin, *β-closure and β-interior*, J. Fac. Ed. Ain Shams Univ. **10** (1986), 235–245.

[3] D. Andrijević, *Semi-preopen sets*, Mat. Vesnik **38** (1986), 24–32.

[4] S. Athisaya Panmani, R. Raja Rajeswari, M. Lellis Thivagar and E. Ekici, *On some characterizations of vividly and blurly (1, 2)-β-irresolute mappings*, Filomat **21** (2007), 85–98.

[5] S. Athisaya Panmani, R. Raja Rajeswari, M. Lellis Thivagar and E. Ekici, *On θ-(1, 2) semi-pregeneralized closed sets*, Filomat **22** (2008), 57–67.

[6] S. G. Crossley and S. K. Hildebrand, *Semi-closure*, Texas J. Sci. **22** (1971), 99–112.

[7] G. Di Maio and T. Noiri, *On s-closed spaces*, Indian J. Pure Appl. Math. **18** (1987), 226–233.

[8] C. Dorsett, *Semi-regular spaces*, Soochow J. Math. **8** (1982), 45–53.

[9] P. C. Dutta, *Contributions of the Theory of Bitopological Spaces*, Ph. D. Thesis, Pilan (India), 1971.

[10] S. N. El-Deeb, I. A. Hasanein, A. S. Mashhour and T. Noiri, *On p-regular spaces*, Bull. Math. Soc. Sci. Math. R. S. Roumanie **27(75)** (1983), 311–315.

[11] M. Lellis Thivagar, *Generalizations on pairwise α-continuous functions*, Pure Appl. Math. Sci. **28** (1991), 53–63.

[12] M. Lellis Thivagar, R. Raja Rajeswari and E. Ekici, *On extension of semi-preopen sets in bitopological spaces*, Kochi J. Math. **3** (2008), 55–60.

[13] N. Levine, *A decomposition of continuity in topological spaces,* Amer. Math. Monthly **68** (1961), 44–46.

[14] N. Levine, *Semi-open sets and semi-continuity in topological spaces,* Amer. Math. Monthly **70** (1963), 36–41.

[15] S. N. Maheshwari, P. C. Jain and G. I. Chae, *On quasi open sets*, Ulsan Inst. Tech. Report **11** (1980), 291–292.

[16] H. Maki, K. C. Rao and A. Nagoor Gani, *On generalizing semi-open and preopen sets*, Pure Appl. Math. Sci. **49** (1999), 17–29.

[17] A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deep, *On precontinuous and weak precontinuous mappings,* Proc. Math. Phys. Soc. Egypt **53** (1982), 47–53.

[18] A. S. Mashhour, I. A. Hasanein and S. N. El-Deeb, *α-continuous and α-open mappings*, Acta Math. Hungar. **41** (1983), 213–218.

[19] G. B. Navalagi, M. Lellis Thivagar and R. Raja Rajeswari, *Generalized semi-preclosed sets in bitopological spaces*, Math. Forum **27** (2004/2005).

[20] O. Njåstad, *On some classes of nearly open sets,* Pacific J. Math. **15** (1965), 961–970.

[21] T. Noiri and V. Popa, *Minimal structures, weakly irresolute functions and bitopological spaces,* Stud. Cerc. St. Ser. Mat. Univ. Bacău **18** (2008), 181–192.

[22] T. Noiri and V. Popa, *On some forms of weakly continuous functions in bitopological spaces,* Demonstratio Math. **41** (2008), 687–700.

[23] M. C. Pal and P. Bhattacharyya, *Feeble and strong forms of preirresolute functions*, Bull. Malaysian Math. Soc. (2) **19** (1996), 63–75.

[24] V. Popa and T. Noiri, *On M-continuous functions,* Anal. Univ. "Dunarea de Jos" Galati, Ser. Mat. Fiz. Mec. Teor., Fasc. II, **18** (**23**) (2000), 31–41.

[25] V. Popa and T. Noiri, *On the definitions of some generalized forms of continuity under minimal conditions*, Mem. Fac. Sci. Kochi Univ. Ser. Math. **22** (2001), 31–41.

[26] V. Popa and T. Noiri, *A unified theory of weak continuity for functions,* Rend Circ. Mat. Palermo (2) **51** (2002), 439–464.

[27] R. Raja Rajeswari and M. Lellis Thivagar, *On extension of semipreopen sets in bitopological spaces*, Proc. Nat. Conf. Pure Appl. Math. (2005), 28–32.

[28] O. Ravi and M. Lellis Thivagar, *A bitopological* $(1,2)^*$*-semigeneralized continuous maps*, Bull. Malays. Math. Sci. Soc. (2) **29** (2006), 1–9.

[29] M. S. Sarsak, *On quasi-continuous functions,* J. Indian Acad. Math. **27**(2006), 407–414.

[30] N. V. Veličko, *H-closed topological spaces,* Amer. Math. Soc. Transl. (2) **78** (1968), 103–118.