A Generalization of Blurly (1,2)-$\beta$-Irresolute Functions

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Abstract. In this paper we introduce the notion of weakly (1,2)-$M$-continuous functions as functions from a set satisfying some minimal conditions into a set satisfying some minimal conditions. We obtain some characterizations and several properties of such functions. This function leads to the formulation of a generalization of blurly (1,2)-$\beta$-irresolute functions defined in [4].

Keywords: $m$-structure, $m$-space, weakly $M$-continuous, (1,2)-semi-preopen, blurly (1,2)-$\beta$-irresolute.

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1 Introduction

Semi-open sets, preopen sets, $\alpha$-open sets and $\beta$-open sets play an important role in the researching of generalizations of continuity in topological spaces and bitopological spaces. By using these sets many authors introduced and studied various types of modifications of continuity in topological spaces and bitopological spaces. The notions of quasi-open sets or $\tau_1\tau_2$-open sets and quasi-continuity or $\tau_1\tau_2$-continuity are studied in [9], [15] and [29]. The notions of (1,2)-semi-open sets, (1,2)-preopen sets, and (1,2)-$\alpha$-open sets and also (1,2)-semi-continuity, (1,2)-precontinuity, and (1,2)-$\alpha$-continuity are introduced in [11]. The notions of (1,2)-semi-preopen sets and (1,2)-semi-precontinuity are introduced and studied in [12] and [27]. In 1961, Levine [13] introduced the concept of weakly continuous functions in topological spaces. By using the notions of semi-open sets, preopen sets, $\alpha$-sets and $\beta$-open sets, several authors introduced and studied various forms of weak continuity in topological spaces and bitopological spaces. Quite recently, a new form of weak continuity is introduced and studied in [4].

In [24], [25] and [26], the present authors introduced and investigated the notions of minimal structures, $m$-spaces, $M$-continuity and weak $M$-continuity. By using the notions of minimal structures and $m$-spaces, the present authors [21], [22] introduced and investigated new forms of weak continuity in bitopological spaces. The purpose of this paper is to introduce the notions of weakly (1,2)-$M$-continuous functions which generalize the notions of blurly (1,2)-$\beta$-irresolute functions [4] and to transfer the study of weakly (1,2)-$M$-continuous functions between bitopological spaces to the study of weakly $M$-continuous functions between $m$-spaces.
2 Preliminaries

Let \((X, \tau)\) be a topological space and \(A\) a subset of \(X\). The closure of \(A\) and the interior of \(A\) are denoted by \(\text{Cl}(A)\) and \(\text{Int}(A)\), respectively.

**Definition 2.1.** Let \((X, \tau)\) be a topological space. A subset \(A\) of \(X\) is said to be

1. semi-open \([14]\) if \(A \subset \text{Cl}(\text{Int}(A))\),
2. preopen \([17]\) if \(A \subset \text{Int}(\text{Cl}(A))\),
3. \(\alpha\)-open \([20]\) if \(A \subset \text{Int}(\text{Cl}(\text{Int}(A)))\),
4. \(\beta\)-open \([1]\) or semi-preopen \([3]\) if \(A \subset \text{Cl}(\text{Int}(\text{Cl}(A)))\).

The family of all semi-open (resp. preopen, \(\alpha\)-open, \(\beta\)-open) sets in \((X, \tau)\) is denoted by \(\text{SO}(X)\) (resp. \(\text{PO}(X)\), \(\alpha(X)\), \(\beta(X)\) or \(\text{SPO}(X)\)).

**Definition 2.2.** The complement of a semi-open (resp. preopen, \(\alpha\)-open, \(\beta\)-open) set is said to be semi-closed \([6]\) (resp. preclosed \([17]\), \(\alpha\)-closed \([18]\), \(\beta\)-closed \([1]\) or semi-preclosed \([3]\)).

**Definition 2.3.** The intersection of all semi-closed (resp. preclosed, \(\alpha\)-closed, \(\beta\)-closed) sets of \(X\) containing \(A\) is called the semi-closure \([6]\) (resp. preclosure \([10]\), \(\alpha\)-closure \([18]\), \(\beta\)-closure \([2]\) or semi-preclosure \([3]\)) of \(A\) and is denoted by \(s\text{Cl}(A)\) (resp. \(p\text{Cl}(A)\), \(\alpha\text{Cl}(A)\), \(\beta\text{Cl}(A)\) or \(s\text{pCl}(A)\)).

**Definition 2.4.** The union of all semi-open (resp. preopen, \(\alpha\)-open, \(\beta\)-open) sets of \(X\) contained in \(A\) is called the semi-interior (resp. preinterior, \(\alpha\)-interior, \(\beta\)-interior or semi-preinterior) of \(A\) and is denoted by \(s\text{Int}(A)\) (resp. \(p\text{Int}(A)\), \(\alpha\text{Int}(A)\), \(\beta\text{Int}(A)\) or \(s\text{pInt}(A)\)).

Throughout the present paper, \((X, \tau)\) and \((Y, \sigma)\) (or simply \(X\) and \(Y\)) denote topological spaces and \((X, \tau_1, \tau_2)\) and \((Y, \sigma_1, \sigma_2)\) denote bitopological spaces.

3 Minimal structures and \(M\)-continuity

**Definition 3.1.** A subfamily \(m_X\) of the power set \(\mathcal{P}(X)\) of a nonempty set \(X\) is called a minimal structure (briefly \(m\)-structure) on \(X\) \([24], [25]\) if \(\emptyset \in m_X\) and \(X \in m_X\).

By \((X, m_X)\), we denote a nonempty subset \(X\) with a minimal structure \(m_X\) on \(X\) and call it an \(m\)-space. Each member of \(m_X\) is said to be \(m_X\)-open (briefly \(m\)-open) and the complement of an \(m_X\)-open set is said to be \(m_X\)-closed (briefly \(m\)-closed).

**Remark 3.1.** Let \((X, \tau)\) be a topological space. Then the families \(\tau, \text{SO}(X), \text{PO}(X), \alpha(X), \beta(X), \text{SPO}(X)\) are all \(m\)-structures on \(X\).

**Definition 3.2.** Let \(X\) be a nonempty set and \(m_X\) an \(m\)-structure on \(X\). For a subset \(A\) of \(X\), the \(m_X\)-closure of \(A\) and the \(m_X\)-interior of \(A\) are defined in \([16]\) as follows:

1. \(m\text{Cl}(A) = \bigcap\{F : A \subset F, X \setminus F \in m_X\}\),
2. \(m\text{Int}(A) = \bigcup\{U : U \subset A, U \in m_X\}\).
Theorem 3.1. Let \( (X, \tau) \) be a topological space and \( A \) a subset of \( X \). If \( m_X = \tau \) (resp. \( SO(X), PO(X), \alpha(X), \beta(X), SPO(X) \)), then we have

\begin{enumerate}
\item \( m\text{Cl}(A) = \text{Cl}(A) \) (resp. \( m\text{Cl}(A), p\text{Cl}(A), \alpha\text{Cl}(A), \beta\text{Cl}(A), sp\text{Cl}(A) \)),
\item \( m\text{Int}(A) = \text{Int}(A) \) (resp. \( s\text{Int}(A), p\text{Int}(A), \alpha\text{Int}(A), \beta\text{Int}(A), sp\text{Int}(A) \)).
\end{enumerate}

Lemma 3.1. (Maki [16]) Let \( X \) be a nonempty set and \( m_X \) an \( m \)-structure on \( X \). For subsets \( A \) and \( B \) of \( X \), the following properties hold:

\begin{enumerate}
\item \( m\text{Cl}(X \setminus A) = X \setminus m\text{Int}(A) \) and \( m\text{Int}(X \setminus A) = X \setminus m\text{Cl}(A) \),
\item If \( X \setminus A \in m_X \), then \( m\text{Cl}(A) = A \) and if \( A \in m_X \), then \( m\text{Int}(A) = A \),
\item \( m\text{Cl}(\emptyset) = \emptyset, m\text{Cl}(X) = X, m\text{Int}(\emptyset) = \emptyset \) and \( m\text{Int}(X) = X \),
\item If \( A \subset B \), then \( m\text{Cl}(A) \subset m\text{Cl}(B) \) and \( m\text{Int}(A) \subset m\text{Int}(B) \),
\item \( A \subset m\text{Cl}(A) \) and \( m\text{Int}(A) \subset A \),
\item \( m\text{Cl}(m\text{Cl}(A)) = m\text{Cl}(A) \) and \( m\text{Int}(m\text{Int}(A)) = m\text{Int}(A) \).
\end{enumerate}

Lemma 3.2. (Popa and Noiri [24]) Let \( X \) be a nonempty set and \( m_X \) an \( m \)-structure on \( X \). Then \( x \in m\text{Cl}(A) \) if and only if \( U \cap A \neq \emptyset \) for every \( U \in m_X \) containing \( x \).

Definition 3.3. An \( m \)-structure \( m_X \) on a nonempty set \( X \) is said to have property \( B \) [16] if the union of any family of subsets belonging to \( m_X \) belongs to \( m_X \).

Lemma 3.3. (Popa and Noiri [26]) Let \( X \) be a nonempty set and \( m_X \) an \( m \)-structure on \( X \) satisfying property \( B \). For a subset \( A \) of \( X \), the following properties hold:

\begin{enumerate}
\item \( A \in m_X \) if and only if \( m\text{Int}(A) = A \),
\item \( A \) is \( m_X \)-closed if and only if \( m\text{Cl}(A) = A \),
\item \( m\text{Int}(A) \in m_X \) and \( m\text{Cl}(A) \) is \( m_X \)-closed.
\end{enumerate}

Definition 3.4. A function \( f : (X, m_X) \to (Y, m_Y) \) is said to be weakly \( M \)-continuous [26] (resp. \( M \)-continuous [24]) at \( x \in X \) if for each \( V \in m_Y \) containing \( f(x) \), there exists \( U \in m_X \) containing \( x \) such that \( f(U) \subset m\text{Cl}(V) \) (resp. \( f(U) \subset V \)). A function \( f : (X, m_X) \to (Y, m_Y) \) is said to be weakly \( M \)-continuous (resp. \( M \)-continuous) if it has the property at each point \( x \in X \).

Theorem 3.1. For a function \( f : (X, m_X) \to (Y, m_Y) \), where \( m_Y \) has property \( B \), the following properties are equivalent:

\begin{enumerate}
\item \( f \) is weakly \( M \)-continuous at \( x \in X \);
\item for every \( V \in m_Y \) with \( x \in f^{-1}(V) \), \( x \in m\text{Int}(f^{-1}(m\text{Cl}(V))) \);
\item for every \( m_Y \)-closed set \( F \) of \( Y \) with \( x \in m\text{Cl}(f^{-1}(m\text{Int}(F))) \), \( x \in f^{-1}(F) \);
\item for every subset \( B \) of \( Y \) with \( x \in m\text{Cl}(f^{-1}(m\text{Int}(m\text{Cl}(B)))) \), \( x \in f^{-1}(m\text{Cl}(B)) \);
\item for every subset \( B \) of \( Y \) with \( x \in f^{-1}(m\text{Int}(B)) \), \( x \in m\text{Int}(f^{-1}(m\text{Cl}(m\text{Int}(B)))) \);
\item for every \( m_Y \)-open set \( V \) with \( x \in m\text{Cl}(f^{-1}(V)) \), \( x \in f^{-1}(m\text{Cl}(V)) \).
\end{enumerate}

Proof. (1) \( \Rightarrow \) (2): Let \( f \) be weakly \( M \)-continuous at \( x \) and \( V \in m_Y \) such that \( x \in f^{-1}(V) \). Then, \( f(x) \in V \) and there exists \( U \in m_X \) containing \( x \) such that \( f(U) \subset m\text{Cl}(V) \). Then we have \( x \in U \subset f^{-1}(m\text{Cl}(V)) \). This implies that \( x \in m\text{Int}(f^{-1}(m\text{Cl}(V))) \).

(2) \( \Rightarrow \) (3): Let \( F \) be any \( m_Y \)-closed set of \( Y \). Suppose that \( x \notin f^{-1}(F) \). Then \( Y \setminus F \in m_Y \) and \( x \in X \setminus f^{-1}(F) = f^{-1}(Y \setminus F) \). By (2) and Lemma 3.1, we have

\[ x \in m\text{Int}(f^{-1}(m\text{Cl}(Y \setminus F))) = m\text{Int}(f^{-1}(Y \setminus m\text{Int}(F))) = m\text{Int}(X \setminus f^{-1}(m\text{Int}(F))) = X \setminus m\text{Cl}(f^{-1}(m\text{Int}(F))). \]
Therefore, we obtain \( x \notin \operatorname{mCl}(f^{-1}((\operatorname{mInt}(F))). \)

(3) \( \Rightarrow \) (4): Let \( B \) be any subset of \( Y \). Since \( m_Y \) has property \( B \), \( \operatorname{mCl}(B) \) is \( m_Y \)-closed and for any \( x \in \operatorname{mCl}(f^{-1}(\operatorname{mInt}(\operatorname{mCl}(B)))), x \in f^{-1}(\operatorname{mCl}(B)). \)

(4) \( \Rightarrow \) (5): Let \( B \) be any subset of \( Y \) and \( x \in f^{-1}(\operatorname{mInt}(B)) \). Then we have \( x \in f^{-1}(\operatorname{mInt}(B)) = X \setminus f^{-1}(\operatorname{mCl}(Y \setminus B)). \) Then \( x \notin f^{-1}(\operatorname{mCl}(Y \setminus B)). \) Therefore, by (4) \( x \in X \setminus \operatorname{mCl}(f^{-1}(\operatorname{mInt}(\operatorname{mCl}(Y \setminus B)))) = \operatorname{mInt}(f^{-1}(\operatorname{mCl}(\operatorname{mInt}(B)))). \)

(5) \( \Rightarrow \) (6): Let \( V \) be any \( m_Y \)-open set of \( Y \). Suppose that \( x \notin f^{-1}(\operatorname{mCl}(V)). \) Then \( f(x) \notin \operatorname{mCl}(V) \) and by Lemma 3.2 there exists \( W \in m_Y \) containing \( f(x) \) such that \( W \cap V = \emptyset \); hence \( \operatorname{mCl}(W) \cap V = \emptyset \). By (5), we have \( x \in \operatorname{mInt}(f^{-1}(\operatorname{mCl}(W))) \) and hence there exists \( U \in m_X \) such that \( x \in U \subset f^{-1}(\operatorname{mCl}(W)). \) Since \( \operatorname{mCl}(W) \cap V = \emptyset \), \( U \cap f^{-1}(V) = \emptyset \) and by Lemma 3.2 \( x \notin \operatorname{mCl}(f^{-1}(V)). \) Therefore, for each \( x \in \operatorname{mCl}(f^{-1}(V)), x \notin f^{-1}(\operatorname{mCl}(V)). \)

(6) \( \Rightarrow \) (1): Let \( V \) be any \( m_Y \)-open set containing \( f(x) \). Then we have \( x \in f^{-1}(V) \subset f^{-1}(\operatorname{mInt}(\operatorname{mCl}(V))) = X \setminus f^{-1}(\operatorname{mCl}(Y \setminus \operatorname{mCl}(V))). \) By (6), \( x \notin \operatorname{mCl}(f^{-1}(Y \setminus \operatorname{mCl}(V))) \) and \( x \in \operatorname{mInt}(f^{-1}(\operatorname{mCl}(V))). \) Therefore, there exists \( U \in m_X \) such that \( x \in U \subset f^{-1}(\operatorname{mCl}(V)); \) hence \( f(U) \subset \operatorname{mCl}(V). \) This shows that \( f \) is weakly \( M \)-continuous at \( x \in X. \)

**Definition 3.5.** Let \( S \) be a subset of an \( m \)-space \((X, m_X)\). A point \( x \in X \) is called an \( m_\theta \)-adherent point of \( S \) if \( \operatorname{mCl}(U) \cap S \neq \emptyset \) for every \( m_X \)-open set \( U \) containing \( x. \)

The set of all \( m_\theta \)-adherent points of \( S \) is called the \( m_\theta \)-closure of \( S \) and is denoted by \( \operatorname{mCl}_\theta(S) \). If \( S = \operatorname{mCl}_\theta(S) \), then \( S \) is said to be \( m_\theta \)-closed. The complement of an \( m_\theta \)-closed set is said to be \( m_\theta \)-open.

**Remark 3.3.** Let \( S \) be a subset of a topological space \((X, \tau)\) and \( m_X = \tau \) (resp. \( \operatorname{SO}(X), \operatorname{PO}(X) \)), then \( \operatorname{mCl}_\theta(S) = \operatorname{Cl}_\theta(S) \) [30] (resp. \( \operatorname{sCl}_\theta(S) \) [7], \( \operatorname{pCl}_\theta(S) \) [23]).

**Lemma 3.4.** (Popa and Noiri [26]) Let \( A \) be a subset of an \( m \)-space \((X, m_X)\). Then the following properties hold:

1. If \( A \) is \( m_X \)-open in \((X, m_X)\), then \( \operatorname{mCl}(A) = \operatorname{mCl}_\theta(A), \)
2. If \( m_X \) satisfies property \( B \), then \( \operatorname{mCl}_\theta(A) \) is \( m_X \)-closed for every subset \( A \) of \( X \).

**Theorem 3.2.** (Popa and Noiri [26], [21]) For a function \( f : (X, m_X) \rightarrow (Y, m_Y) \), the following properties are equivalent:

1. \( f \) is weakly \( M \)-continuous;
2. \( f^{-1}(V) \subset \operatorname{mInt}(f^{-1}(\operatorname{mCl}(V))) \) for every \( V \in m_Y; \)
3. \( f(\operatorname{mCl}(A)) \subset \operatorname{mCl}(f(A)) \) for every subset \( A \) of \( X; \)
4. \( \operatorname{mCl}(f^{-1}(B)) \subset f^{-1}(\operatorname{mCl}_\theta(B)) \) for every subset \( B \) of \( Y; \)
5. \( \operatorname{mCl}(f^{-1}(\operatorname{mInt}(\operatorname{mCl}(B)))) \subset f^{-1}(\operatorname{mCl}(B)) \) for every subset \( B \) of \( Y; \)
6. \( \operatorname{mCl}(f^{-1}(\operatorname{mInt}(F))) \subset f^{-1}(F) \) for every \( m_Y \)-closed set \( F \) of \( Y. \)

**Theorem 3.3.** (Popa and Noiri [26]) Let \((Y, m_Y)\) be an \( m \)-space and \( m_Y \) satisfy \( B \). Then, a function \( f : (X, m_X) \rightarrow (Y, m_Y) \) is weakly \( M \)-continuous if and only if \( \operatorname{mCl}(f^{-1}(V)) \subset f^{-1}(\operatorname{mCl}(V)) \) for every \( m_Y \)-open set \( V \) of \( Y. \)

For a function \( f : (X, m_X) \rightarrow (Y, m_Y) \), we define \( D_{WMC}(f) \) as follows:

\[
D_{WMC}(f) = \{ x \in X : f \text{ is not weakly } M\text{-continuous at } x \}. \]
Theorem 3.4. For a function \( f : (X, m_X) \to (Y, m_Y) \), where \( m_Y \) has property \( \mathcal{B} \), the following equalities hold:

\[
D_{WMC}(f) = \bigcup_{G \in m_Y} \{ f^{-1}(G) \setminus \text{mInt}(f^{-1}(\text{mCl}(G))) \} \\
= \bigcup_{H \in \mathcal{F}} \{ \text{mCl}(f^{-1}(\text{mInt}(H))) \setminus f^{-1}(H) \} \\
= \bigcup_{B \in \mathcal{P}(Y)} \{ \text{mCl}(f^{-1}(\text{mInt}(\text{mCl}(B)))) \setminus f^{-1}(\text{mCl}(B)) \} \\
= \bigcup_{B \in \mathcal{P}(Y)} \{ f^{-1}(\text{mInt}(B)) \setminus \text{mInt}(f^{-1}(\text{mCl}(B))) \} \\
= \bigcup_{G \in m_Y} \{ \text{mCl}(f^{-1}(G)) \setminus f^{-1}(\text{mCl}(G)) \},
\]

where \( \mathcal{F} \) is the family of \( m_Y \)-closed sets of \( (Y, m_Y) \).

Proof. We shall show only the first equality since the proofs of other are similar to the first. Let \( x \in D_{WMC}(f) \). By Theorem 3.1, there exists \( V \in m_Y \) such that \( x \in f^{-1}(V) \) and \( x \notin \text{mInt}(f^{-1}(\text{mCl}(V))) \). Therefore, we have

\[
x \in f^{-1}(V) \setminus \text{mInt}(f^{-1}(\text{mCl}(V))) \subset \bigcup_{G \in m_Y} \{ f^{-1}(G) \setminus \text{mInt}(f^{-1}(\text{mCl}(G))) \}.
\]

Conversely, let \( x \in \bigcup_{G \in m_Y} \{ f^{-1}(G) \setminus \text{mInt}(f^{-1}(\text{mCl}(G))) \} \). Then, there exists \( V \in m_Y \) such that \( x \in f^{-1}(V) \setminus \text{mInt}(f^{-1}(\text{mCl}(V))) \). By Theorem 3.1, we obtain \( x \in D_{WMC}(f) \).

Definition 3.6. Let \( (X, m_X) \) be an \( m \)-space and \( A \) a subset of \( X \). The \( m \)-frontier of \( A \), \( m\text{Fr}(A) \), is defined as follows: \( m\text{Fr}(A) = m\text{Cl}(A) \cap m\text{Cl}(X \setminus A) = m\text{Cl}(A) \setminus m\text{Int}(A) \).

Theorem 3.5. (Popa and Noiri [26]) The set of all points \( x \in X \) at which a function \( f : (X, m_X) \to (Y, m_Y) \) is not weakly \( M \)-continuous is identical with the union of the \( m \)-frontiers of the inverse images of the \( m_Y \)-closures of \( m_Y \)-open sets containing \( f(x) \).

4 Minimal structures and bitopological spaces

Definition 4.1. A subset \( A \) of a bitopological space \( (X, \tau_1, \tau_2) \) is said to be quasi-open [9], [29] or \( \tau_1\tau_2 \)-open [11] if \( A = B \cup C \), where \( B \in \tau_1 \) and \( C \in \tau_2 \).

The complement of a \( \tau_1\tau_2 \)-open set is said to be \( \tau_1\tau_2 \)-closed. The intersection of all \( \tau_1\tau_2 \)-closed sets containing a subset \( A \) of \( X \) is called the \( \tau_1\tau_2 \)-closure and is denoted by \( \tau_1\tau_2\text{Cl}(A) \). The union of all \( \tau_1\tau_2 \)-open sets of \( X \) contained in \( A \) is called the \( \tau_1\tau_2 \)-interior of \( A \) and is denoted by \( \tau_1\tau_2\text{Int}(A) \). The family of all \( \tau_1\tau_2 \)-open sets of \( (X, \tau_1, \tau_2) \) is denoted by \( (1,2)\text{O}(X) \).

Definition 4.2. A subset \( A \) of a bitopological space \( (X, \tau_1, \tau_2) \) is said to be

1. \( (1,2) \)-semi-open [11] if \( A \subset \tau_1\tau_2\text{Cl}(\tau_1\text{Int}(A)) \),
2. \( (1,2) \)-preopen [11] if \( A \subset \tau_1\text{Cl}(\tau_1\text{Cl}(A)) \),
3. \( (1,2) \)-\( \alpha \)-open [11] if \( A \subset \tau_1\text{Int}(\tau_1\tau_2\text{Cl}(\tau_1\text{Int}(A))) \),
4. \( (1,2) \)-semi-preopen [12], [27] if \( A \subset \tau_1\tau_2\text{Cl}(\tau_1\text{Int}(\tau_1\tau_2\text{Cl}(A))) \).
The complement of $(1,2)$-semi-open (resp. $(1,2)$-preopen, $(1,2)$-α-open, $(1,2)$-semi-preopen) set of $X$ is said to be $(1,2)$-semi-closed (resp. $(1,2)$-preclosed, $(1,2)$-α-closed, $(1,2)$-semi-preclosed). The intersection of all $(1,2)$-semi-closed (resp. $(1,2)$-preclosed, $(1,2)$-α-closed, $(1,2)$-semi-preclosed) sets containing $A$ is called the $(1,2)$-semi-closure (resp. $(1,2)$-preclosure, $(1,2)$-α-closure, $(1,2)$-semi-preclosure) of $A$ and is denoted by $(1,2)sCl(A)$ (resp. $(1,2)pCl(A), (1,2)αCl(A), (1,2)spCl(A)$). The union of $(1,2)$-semi-open (resp. $(1,2)$-preopen, $(1,2)$-α-open, $(1,2)$-semi-preopen) sets of $X$ contained in $A$ is called the $(1,2)$-semi-interior (resp. $(1,2)$-preinterior, $(1,2)$-α-interior, $(1,2)$-semi-preinterior) of $A$ and is denoted by $(1,2)sInt(A)$ (resp. $(1,2)pInt(A), (1,2)αInt(A), (1,2)spInt(A)$).

The collection of all $(1,2)$-semi-open (resp. $(1,2)$-preopen, $(1,2)$-α-open, $(1,2)$-semi-preopen) sets of $X$ is denoted by $(1,2)SO(X)$ (resp. $(1,2)PO(X), (1,2)αO(X), (1,2)SPO(X)$).

**Remark 4.1.** Let $(X, τ_1, τ_2)$ be a bitopological space and $A$ a subset of $X$.

1. $(1) τ_1 T_2 O(X), (1,2)SO(X), (1,2)PO(X), (1,2)αO(X)$ and $(1,2)SPO(X)$ are all $m$-structures on $X$ having property $E$.

2. By $(1,2)M(X)$, we denote the collection of all $m$-structures determined by the topologies $τ_1$ and $τ_2$ on $X$ and call each member of $(1,2)M(X)$ a $(1,2)M$-structure on $X$. Let $(1,2)M(X) ∈ (1,2)M(X)$. If $(1,2)M(X) = τ_1 T_2 O(X)$ (resp. $(1,2)SO(X), (1,2)PO(X), (1,2)αO(X), (1,2)SPO(X)$), then we have
   
   i) $(1,2)mCl(A) = τ_1 T_2 Cl(A)$ (resp. $(1,2)sCl(A), (1,2)pCl(A), (1,2)αCl(A), (1,2)spCl(A)$),
   
   ii) $(1,2)mInt(A) = τ_1 T_2 Int(A)$ (resp. $(1,2)sInt(A), (1,2)pInt(A), (1,2)αInt(A), (1,2)spInt(A)$).

3. A subset $A$ is $τ_1 T_2$-closed (resp. $(1,2)$-semi-closed, $(1,2)$-preclosed, $(1,2)$-α-closed, $(1,2)$-semi-preclosed) if and only if $A = τ_1 T_2 Cl(A)$ (resp. $A = (1,2)sCl(A), A = (1,2)pCl(A), A = (1,2)αCl(A)$).

4. $A$ is $τ_1 T_2$-open (resp. $(1,2)$-semi-open, $(1,2)$-preopen, $(1,2)$-α-open, $(1,2)$-semi-preopen) if and only if $A = τ_1 T_2 Int(A)$ (resp. $A = (1,2)sInt(A), A = (1,2)pInt(A), A = (1,2)αInt(A)$).

5. By Lemma 3.2, we obtain the results established in Lemma 8(iii) of [4].

6. By Lemma 3.1, we obtain the relations between $(1,2)mCl(A)$ and $(1,2)mInt(A)$.

7. Let $(1,2)M(X) ∈ (1,2)M(X)$ and $A$ be a subset of $X$. By $(1,2)mCl_0(A)$, we denote the $m_0$-closure of $A$ in $(1,2)M(X)$.

5 Weakly $(1,2)$-$M$-continuous functions

**Definition 5.1.** A function $f : (X, τ_1, τ_2) → (Y, σ_1, σ_2)$ is said to be **blurly $(1,2)$-β-irresolute** [4] if for each point $x ∈ X$ and each $V ∈ (1,2)SPO(Y)$ containing $f(x)$, there exists $U ∈ (1,2)SPO(X)$ containing $x$ such that $f(U) ⊂ (1,2)spCl(V)$.

Hence, it turns out that $f : (X, τ_1, τ_2) → (Y, σ_1, σ_2)$ is blurly $(1,2)$-β-irresolute if and only if $f : (X, (1,2)SPO(X)) → (Y, (1,2)SPO(Y))$ is weakly $M$-continuous.

**Definition 5.2.** Let $(X, τ_1, τ_2)$ and $(Y, σ_1, σ_2)$ be bitopological spaces and $(1,2)M(X)$ (resp. $(1,2)M(Y)$) a $(1,2)M$-structure on $X$ (resp. $Y$). A function $f : (X, τ_1, τ_2) → (Y, σ_1, σ_2)$ **is weakly $(1,2)$-$M$-continuous** if for each $A ⊆ X$ and each $V ∈ (1,2)SPO(Y)$ containing $f(A)$, there exists $U ⊆ X$ such that $f(U) ⊂ (1,2)spCl(V)$. 
is stated in (1) and (2).

Remark 5.1. (1) If \((1,2)\)\(M(X) = (1,2)\)\(SPO(X)\) and \((1,2)\)\(M(Y) = (1,2)\)\(SPO(Y)\), then we obtain the definitions of \((1,2)\)-\(\beta\)-irresolute functions \([19]\) and blurly \((1,2)\)-\(\beta\)-irresolute functions \([4]\).

(2) If \((1,2)\)\(M(X) = (1,2)\)\(O(X)\) (resp. \((1,2)\)\(SO(X)\), \((1,2)\)\(PO(X)\), \((1,2)\)\(O(X)\)), \((1,2)\)\(O(Y)\) (resp. \((1,2)\)\(SO(Y)\), \((1,2)\)\(PO(Y)\), \((1,2)\)\(O(Y)\), \((1,2)\)\(SPO(Y)\)) and \(f : (X, (1,2)\)\(M(X)\)) \(\rightarrow\) \((Y, (1,2)\)\(M(Y)\)) is \((1,2)\)-continuous or weakly \((1,2)\)-\(M\)-continuous, then we obtain the definitions of \((1,2)\)-\(\beta\)-continuity (or weak \((1,2)\)-\(M\)-irresolute) function or a weakly \((1,2)\)-\(M\)-continuous, \((1,2)\)-\(\alpha\)-continuous, \((1,2)\)-\(\alpha\)-precontinuous, weakly \((1,2)\)-\(M\)-continuous, weakly \((1,2)\)-\(\alpha\)-continuous, weakly \((1,2)\)-\(\alpha\)-precontinuous function.

(3) \((1,2)\)-\(M\)\(-\)continuity (or weak \((1,2)\)-\(M\)-continuity) is a unified form of the functions stated in (1) and (2).

The following three theorems follow from Theorems 3.1, 3.2 and 3.3.

Theorem 5.1. Let \((X, \tau_1, \tau_2)\) and \((Y, \sigma_1, \sigma_2)\) be bitopological spaces and \((1,2)\)\(M(X)\) (resp. \((1,2)\)\(M(Y)\)) a \((1,2)\)\(M\)-structure on \(X\) (resp. \(Y\)), where \((1,2)\)\(M(Y)\) has property \(B\). For a function \(M : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)\), the following properties are equivalent:

1. \(f\) is weakly \((1,2)\)-\(M\)-continuous at \(x \in X\);
2. for every \((1,2)\)\(M\)\(-\)open set \(V\) of \(Y\) with \(x \in f^{-1}(V), x \in (1,2)\)\(m\)\(\text{Int}(f^{-1}((1,2)\)\(m\)\(\text{Cl}(V)))\);
3. for every \((1,2)\)\(M\)\(-\)closed set \(F\) of \(Y\) with \(x \in (1,2)\)\(m\)\(\text{Cl}(f^{-1}((1,2)\)\(m\)\(\text{Int}(F)))\), \(x \in f^{-1}(F)\);
4. for every subset \(B\) of \(Y\) with \(x \in (1,2)\)\(m\)\(\text{Cl}(f^{-1}((1,2)\)\(m\)\(\text{Int}(B)))\), \(x \in f^{-1}((1,2)\)\(m\)\(\text{Cl}(B)))\);
5. for every subset \(B\) of \(Y\) with \(x \in f^{-1}((1,2)\)\(m\)\(\text{Int}(B))\), \(x \in (1,2)\)\(m\)\(\text{Int}(f^{-1}((1,2)\)\(m\)\(\text{Cl}(B)))\));
6. for every \((1,2)\)\(M\)\(-\)open set \(V\) with \(x \in (1,2)\)\(m\)\(\text{Cl}(f^{-1}(V))\), \(x \in f^{-1}((1,2)\)\(m\)\(\text{Cl}(V)))\).

Theorem 5.2. Let \((X, \tau_1, \tau_2)\) and \((Y, \sigma_1, \sigma_2)\) be bitopological spaces and \((1,2)\)\(M(X)\) (resp. \((1,2)\)\(M(Y)\)) a \((1,2)\)\(M\)-structure on \(X\) (resp. \(Y\)). For a function \(f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)\), the following properties are equivalent:

1. \(f\) is weakly \((1,2)\)-\(M\)-continuous;
2. \(f^{-1}(V) \subset (1,2)\)\(m\)\(\text{Int}(f^{-1}((1,2)\)\(m\)\(\text{Cl}(V)))\) for every \((1,2)\)\(M\)\(-\)open set \(V\) of \(Y\);
3. \(f((1,2)\)\(m\)\(\text{Cl}(A)) \subset (1,2)\)\(m\)\(\text{Cl}_{\varphi}(f(A))\) for every subset \(A\) of \(X\);
4. \((1,2)\)\(m\)\(\text{Cl}(f^{-1}(B)) \subset f^{-1}((1,2)\)\(m\)\(\text{Cl}(B)))\) for every subset \(B\) of \(Y\);
5. \((1,2)\)\(m\)\(\text{Cl}(f^{-1}((1,2)\)\(m\)\(\text{Int}(B)))) \subset f^{-1}((1,2)\)\(m\)\(\text{Cl}(B)))\) for every subset \(B\) of \(Y\);
6. \((1,2)\)\(m\)\(\text{Cl}(f^{-1}((1,2)\)\(m\)\(\text{Int}(F)))\) \(\subset f^{-1}(F)\) for every \((1,2)\)\(M\)\(-\)closed set \(F\) of \(Y\).

Theorem 5.3. Let \((X, \tau_1, \tau_2)\) and \((Y, \sigma_1, \sigma_2)\) be bitopological spaces and \((1,2)\)\(M(X)\) (resp. \((1,2)\)\(M(Y)\)) a \((1,2)\)\(M\)-structure on \(X\) (resp. \(Y\)), where \((1,2)\)\(M(Y)\) has property \(B\). A function \(f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)\) is weakly \((1,2)\)-\(M\)-continuous if and only if \((1,2)\)\(m\)\(\text{Cl}(f^{-1}(V)) \subset f^{-1}((1,2)\)\(m\)\(\text{Cl}(V)))\) for every \((1,2)\)\(M\)\(-\)open set \(V\) of \(Y\).
Remark 5.2. (1) If \((1,2)M(X) = (1,2)\text{SPO}(X)\) and \((1,2)M(Y) = (1,2)\text{SPO}(Y)\), then by Theorems 5.2 and 5.3 we obtain the results established in Theorem 29 of [4] and Theorem 31 (i), (ii), (iii) of [5].

(2) The results of Theorem 34 of [4] are not correct (see Definition 30 and Theorem 3.5 of [5]).

Let \((X, \tau_1, \tau_2)\) and \((Y, \sigma_1, \sigma_2)\) be bitopological spaces and \((1,2)M(X)\) (resp. \((1,2)M(Y)\)) a \((1,2)M\)-structure on \(X\) (resp. \(Y\)). For a function \(f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)\), we define \(D_{(1,2)WMC}(f)\) as follows:

\[ D_{(1,2)WMC}(f) = \{ x \in X : f \text{ is not weakly } (1,2)M\text{-continuous at } x \}. \]


6 Weak \((1,2)M\)-continuity and \((1,2)M\)-continuity

In this section, we investigate the relationships between \((1,2)M\)-continuous functions and weakly \((1,2)M\)-continuous functions.

Definition 6.1. A function \(f : (X, m_X) \to (Y, m_Y)\) is said to satisfy the \(m\)-interiority condition [26] if \(m\text{Int}(f^{-1}(m\text{Cl}(V))) \subseteq f^{-1}(V)\) for each \(m_Y\)-open set \(V\) of \(Y\).

Lemma 6.1. (Popa and Noiri [26]) If a function \(f : (X, m_X) \to (Y, m_Y)\) is weakly \(M\)-continuous and satisfies \(m\)-interiority condition, then \(f\) is \(M\)-continuous.
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**Definition 6.2.** Let \((X, \tau_1, \tau_2)\) and \((Y, \sigma_1, \sigma_2)\) be bitopological spaces and \((1,2)M(X)\) (resp. \((1,2)M(Y)\)) a \((1,2)M\)-structure on \(X\) (resp. \(Y\)). A function \(f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)\) is said to satisfy \((1,2)M\)-interiority condition if \(f : (X, (1,2)M(X)) \to (Y, (1,2)M(Y))\) satisfies \(m\)-interiority condition.

Therefore, a function \(f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)\) satisfies \((1,2)M\)-interiority condition if \((1,2)\text{mlnt}(f^{-1}(1,2)m\text{Cl}(V))) \subset f^{-1}(V)\) for every \((1,2)M\)-open set \(V\) of \(Y\).

**Theorem 6.1.** Let \((X, \tau_1, \tau_2)\) and \((Y, \sigma_1, \sigma_2)\) be bitopological spaces and \((1,2)M(X)\) (resp. \((1,2)M(Y)\)) a \((1,2)M\)-structure on \(X\) (resp. \(Y\)). If \(f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)\) is weakly \((1,2)M\)-continuous and \((1,2)M\)-interiority condition, then \(f\) is \((1,2)M\)-continuous.

**Proof.** The proof is an immediate consequence of Lemma 6.1.

**Definition 6.3.** An \(m\)-space \((X, m_X)\) is said to be \(m\)-regular [26] if for each \(m_X\)-closed set \(F\) and each \(x \notin F\), there exist disjoint \(m_X\)-open sets \(U\) and \(V\) such that \(x \in U\) and \(F \subset V\).

**Remark 6.1.** Let \((X, \tau)\) be a topological space and \(m_X = \tau\) (resp. \(SO(X), PO(X)\)). Then \(m\)-regularity coincides with regularity (resp. semi-regularity [8], pre-regularity [23]).

**Lemma 6.2.** (Popa and Noiri [26]) Let \((Y, m_Y)\) be an \(m\)-regular \(m\)-space and satisfy property \(B\). Then, for a function \(f : (X, m_X) \to (Y, m_Y)\), the following properties are equivalent:

1. \(f\) is \(M\)-continuous;
2. \(f^{-1}(\text{mCl}(B)) = \text{mCl}(f^{-1}(\text{mCl}(B)))\) for every subset \(B\) of \(Y\);
3. \(f\) is weakly \(M\)-continuous;
4. \(f^{-1}(F) = \text{mCl}(f^{-1}(F))\) for every \(m_\theta\)-closed set \(F\) of \(Y\);
5. \(f^{-1}(V) = \text{mlnt}(f^{-1}(V))\) for every \(m_\theta\)-open set \(V\) of \(Y\).

**Definition 6.4.** Let \((X, \tau_1, \tau_2)\) be a bitopological space and \((1,2)M(X)\) a \((1,2)M\)-structure on \(X\). Then \((X, \tau_1, \tau_2)\) is said to be \((1,2)M\)-regular if \((X, (1,2)M(X))\) is \(m\)-regular, equivalently if for each \((1,2)M\)-closed set \(F\) of \((X, \tau_1, \tau_2)\) and each \(x \notin F\), there exist disjoint \((1,2)M\)-open sets \(U\) and \(V\) such that \(x \in U\) and \(F \subset V\).

**Remark 6.2.** If \((X, \tau_1, \tau_2)\) is \((1,2)M\)-regular and \((1,2)M(X) = (1,2)\text{SPO}(X)\), then \((X, \tau_1, \tau_2)\) is \((1,2)\text{-semi-preregular}\ [4].

**Theorem 6.2.** Let \((X, \tau_1, \tau_2)\) and \((Y, \sigma_1, \sigma_2)\) be bitopological spaces and \((1,2)M(X)\) (resp. \((1,2)M(Y)\)) a \((1,2)M\)-structure on \(X\) (resp. \(Y\)), where \((1,2)M(Y)\) satisfies property \(B\) and \((Y, \sigma_1, \sigma_2)\) is \((1,2)M\)-regular. Then, for a function \(f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)\), the following properties are equivalent:

1. \(f\) is \((1,2)M\)-continuous;
2. \(f^{-1}((1,2)m\text{Cl}(B)) = (1,2)m\text{Cl}(f^{-1}((1,2)m\text{Cl}(B)))\) for every subset \(B\) of \(Y\);
3. \(f\) is weakly \((1,2)M\)-continuous;
4. \(f^{-1}(F) = (1,2)m\text{Cl}(f^{-1}(F))\) for every \((1,2)m_\theta\)-closed set \(F\) of \(Y\);
5. \(f^{-1}(V) = (1,2)\text{mlnt}(f^{-1}(V))\) for every \((1,2)m_\theta\)-open set \(V\) of \(Y\).

**Proof.** The proof is an immediate consequence of Lemma 6.2.

**Remark 6.3.** If \((1,2)M(X) = (1,2)\text{SPO}(X)\) and \((1,2)M(Y) = (1,2)\text{SPO}(Y)\), then by Theorem 5.2 we obtain the result established in Theorem 37 of [4].
7 Properties of weakly \((1,2)-M\)-continuous functions

We can obtain several properties of weakly \((1,2)-M\)-continuous functions by using those of weakly \(M\)-continuous functions established in [26].

**Definition 7.1.** An \(m\)-space \((X, m_X)\) is said to be \(m-T_2\) \([24]\) (resp. \(m\)-Urysohn \([26]\)) if for each distinct points \(x, y \in X\), there exist \(U, V \in m_X\) containing \(x\) and \(y\), respectively, such that \(U \cap V = \emptyset\) (resp. \(mCl(U) \cap mCl(V) = \emptyset\)).

**Definition 7.2.** Let \((X, \tau_1, \tau_2)\) be a bitopological space and \((1,2)M(X)\) a \((1,2)\)-\(M\)-structure on \(X\). Then \((X, \tau_1, \tau_2)\) is said to be \((1,2)m-T_2\) (resp. \((1,2)\)-\(M\)-Urysohn) if \((X, (1,2)M(X))\) is \(m-T_2\) (resp. \(m\)-Urysohn).

**Remark 7.1.** If \((X, \tau_1, \tau_2)\) is \((1,2)m-T_2\) and \((1,2)M(X) = (1,2)\text{SPO}(X)\), then \((X, \tau_1, \tau_2)\) is \((1,2)-\beta-T_2\) \([4]\).

**Lemma 7.1.** (Popa and Noiri \([26]\)) Let \(f : (X, m_X) \to (Y, m_Y)\) be a weakly \(M\)-continuous injection and \((Y, m_Y)\) is \(m\)-Urysohn, then \((X, m_X)\) is \(m-T_2\).

**Theorem 7.1.** Let \((X, \tau_1, \tau_2)\) and \((Y, \sigma_1, \sigma_2)\) be bitopological spaces and \((1,2)M(X)\) (resp. \((1,2)M(Y)\)) a \((1,2)\)-\(M\)-structure on \(X\) (resp. \(Y\)). If \(f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)\) is a weakly \((1,2)\)-\(M\)-continuous injection and \((Y, \sigma_1, \sigma_2)\) is \((1,2)m\)-Urysohn, then \((X, \tau_1, \tau_2)\) is \((1,2)m-T_2\).

**Proof.** The proof is an immediate consequence of Lemma 7.1.

**Definition 7.3.** A function \(f : (X, m_X) \to (Y, m_Y)\) is said to have a strongly \(M\)-closed graph \([26]\) if for each \((x, y) \in (X \times Y) - G(f)\), there exist an \(m_X\)-open set \(U\) containing \(x\) and an \(m_Y\)-open set \(V\) containing \(y\) such that \([U \times mCl(V)] \cap G(f) = \emptyset\).

**Definition 7.4.** Let \((X, \tau_1, \tau_2)\) and \((Y, \sigma_1, \sigma_2)\) be bitopological spaces and \((1,2)M(X)\) (resp. \((1,2)M(Y)\)) a \((1,2)\)-\(M\)-structure on \(X\) (resp. \(Y\)). A function \(f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)\) is said to have strongly \((1,2)\)-\(M\)-closed graph if \(f : (X, (1,2)M(X)) \to (Y, (1,2)M(Y))\) has a strongly \(m\)-closed graph.

**Lemma 7.2.** (Popa and Noiri \([26]\)) Let \(f : (X, m_X) \to (Y, m_Y)\) be a weakly \(M\)-continuous function. Then the following properties hold:

1. If \((Y, m_Y)\) is \(m\)-Urysohn, then \(G(f)\) is strongly \(M\)-closed,
2. If \(f\) is injective and \(G(f)\) is strongly \(M\)-closed, then \((X, m_X)\) is \(m-T_2\).

**Theorem 7.2.** Let \((X, \tau_1, \tau_2)\) and \((Y, \sigma_1, \sigma_2)\) be bitopological spaces and \((1,2)M(X)\) (resp. \((1,2)M(Y)\)) a \((1,2)\)-\(M\)-structure on \(X\) (resp. \(Y\)). For a weakly \((1,2)\)-\(M\)-continuous function \(f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)\), the following properties hold:

1. If \((Y, \sigma_1, \sigma_2)\) is \((1,2)m\)-Urysohn, then \(G(f)\) is strongly \((1,2)\)-\(M\)-closed,
2. If \(f\) is injective and \(G(f)\) is strongly \(M\)-closed, then \((X, \tau_1, \tau_2)\) is \((1,2)m-T_2\)

**Proof.** The proof is an immediate consequence of Lemma 7.2.

**Definition 7.5.** An \(m\)-space \((X, m_X)\) is said to be \(m\)-connected \([24]\) if \(X\) cannot be written as the union of two nonempty disjoint \(m_X\)-open sets.
Definition 7.6. Let \((X, \tau_1, \tau_2)\) be a bitopological space and \((1,2)\mathcal{M}(X)\) a \((1,2)\mathcal{M}\)-structure on \(X\). Then \((X, \tau_1, \tau_2)\) is said to be \((1,2)\mathcal{M}\)-connected if \((X, (1,2)\mathcal{M}(X))\) is \(m\)-connected.

Lemma 7.3. (Popa and Noiri [26]) If \(f : (X, m_X) \rightarrow (Y, m_Y)\) is a weakly \(M\)-continuous surjection, where \(m_X\) satisfies \(\mathcal{B}\) and \((X, m_X)\) is \(m\)-connected, then \((Y, m_Y)\) is \(m\)-connected.

Theorem 7.3. Let \((X, \tau_1, \tau_2)\) and \((Y, \sigma_1, \sigma_2)\) be bitopological spaces and \((1,2)\mathcal{M}(X)\) (resp. \((1,2)\mathcal{M}(Y)\)) a \((1,2)\mathcal{M}\)-structure on \(X\) (resp. \(Y\)), where \((1,2)\mathcal{M}(X)\) satisfies property \(\mathcal{B}\). If \(f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)\) is a weakly \((1,2)\)-\(M\)-continuous surjection and \((X, \tau_1, \tau_2)\) is \((1,2)\mathcal{M}\)-connected, then \((Y, \sigma_1, \sigma_2)\) is \((1,2)\mathcal{M}\)-connected.

Proof. The proof is an immediate consequence of Lemma 7.3.

Definition 7.7. A subset \(K\) of an \(m\)-space \((X, m_X)\) is said to be \(m\)-closed [24] (resp. \(m\)-compact [24]) relative to \((X, m_X)\) if for any cover \(\{V_\alpha : \alpha \in \Delta\}\) of \(K\) by \(m_X\)-open sets of \(X\), there exists a finite subset \(\Delta_0\) of \(\Delta\) such that \(K \subset \bigcup\{m\text{Cl}(V_\alpha) : \alpha \in \Delta_0\}\) (resp. \(K \subset \bigcup\{V_\alpha : \alpha \in \Delta_0\}\)). If \(X\) is \(m\)-closed (resp. \(m\)-compact) relative to \((X, m_X)\), then \((X, m_X)\) is said to be \(m\)-closed (resp. \(m\)-compact).

Definition 7.8. Let \((X, \tau_1, \tau_2)\) be a bitopological space and \((1,2)\mathcal{M}(X)\) a \((1,2)\)\(\mathcal{M}\)-structure on \(X\). Then a subset \(K\) of \((X, \tau_1, \tau_2)\) is said to be \((1,2)\mathcal{M}\)-compact, then \(K\) is \((1,2)\mathcal{M}\)-closed relative to \((X, \tau_1, \tau_2)\).

Lemma 7.4. (Popa and Noiri [26]) Let \(f : (X, m_X) \rightarrow (Y, m_Y)\) be a weakly \(M\)-continuous function. Then the following properties hold:

1. If \(K\) is \(m\)-compact relative to \((X, m_X)\), then \(f(K)\) is \(m\)-closed relative to \((Y, m_Y)\),
2. If \(f\) is a surjection and \((X, m_X)\) is \(m\)-compact, then \((Y, m_Y)\) is \(m\)-closed.

Theorem 7.4. Let \(f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)\) be a weakly \((1,2)\)-\(M\)-continuous function and \((1,2)\mathcal{M}(X)\) (resp. \((1,2)\mathcal{M}(Y)\)) a \((1,2)\mathcal{M}\)-structure on \(X\) (resp. \(Y\)). Then the following properties hold:

1. If \(K\) is \((1,2)\mathcal{M}\)-compact relative to \((X, \tau_1, \tau_2)\), then \(f(K)\) is \((1,2)\mathcal{M}\)-closed relative to \((Y, \sigma_1, \sigma_2)\),
2. If \(f\) is a surjection and \((X, \tau_1, \tau_2)\) is \((1,2)\mathcal{M}\)-compact, then \((Y, \sigma_1, \sigma_2)\) is \((1,2)\mathcal{M}\)-closed.

Proof. The proof is an immediate consequence of Lemma 7.4.

References


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