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CORNELIU CONSTANTINESCU AT HIS 70th ANNIVERSARY

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Corneliu Constantinescu is one of those Romanian mathematicians, who in the 2nd half of 20th century strongly influenced the evolution of certain mathematical fields.

His work especially dedicated to the Mathematical Analysis (Riemann Surfaces, Potentialtheory, Measure and Integraltheory) but also to the Topology, Set Theory, Foundations of Mathematics constitutes one of the great achievements in this science. It is characterized by orientation towards important problems which preoccupied at that time the mathematical world and mostly relies on the axiomatic method, which permitted him to construct vast syntheses by introducing new concepts and new points of view.

Born on February 9th, 1929 in Buzau, a town in Romania where he followed the elementary school, he graduated the secondary studies at Collegium "Mihai the Brave" in Bucharest and continued in the fall 1947 with Mathematics at the University of Bucharest, Faculty of Sciences and in the same time he matriculated at the Polytechnic School, Faculty of Constructions . Like for many young people in those ycars, life and plans were totally overturned by the evolution of the political situation in our country. In autumn 1948 he was arrested and condemned to six months correctional imprisonment for plotting against social order. Luckily for mathematics, he was eliberated in January 1950 and could graduate both faculties in 1954. Nevertheless the way to a university career was closed for him. Professor Simion Stoilow, who supervised his licence thesis and greatly appreciated his exceptional gift for mathematics, succeeded only with extreme efforts to obtain his appointment as research mathematician at the Institute of Mathematics of the Romanian Academy. Here he became principal research mathematician in 1958, head of Function Theory subsection in 1963 and head of Function Theory section in 1965.

Though after the 2nd War Romania went through a long tragic period of brutal social transformations, the Romanian mathematics resisted due to the influence of professor Simion Stoilow surrounded by our other great professors Grigore C.Moisil, Miron Nicolescu, Gheorghe Vranceanu, Nicolae Teodorescu. Even more, it had through the foundation of the Institute of Mathematics of the Romanian Academy in 1949 a flourishing development.

In the atmosphere of intense studies and researches of the Institute and especially of the Stoilow Seminar, which took place every Thursday morning, Corneliu Constantinescu acquired a large and solid mathematical culture. Endowed with an exceptional talent, hard working, with a strong power of analysis, abstraction and creation, he had also a good spirit of organization. Of rare correctness and moral distinction, he was ready for scientific friendship and collaboration, and had an active role in the Seminar. Together with Martin Jurchescu, Ionel Bucur, Israel Bernstein, Felix Albrecht, Aristide Deleanu, Petru Caraman myself and other colleagues, he initiated expository cycles on topics of great actuality at that time.

The first papers of Corneliu Constantinescu have their initial point in the special lectures of professor Stoilow as well as in the classic books of Rolf Nevanlinna. Published in 1955 in Romanian language according to regulations then in force, they were republished in international journals due to the relaxation which followed the approval of the organization in Bucharest in 1956 of the IV Congress of Romanian Mathematicians, held with a brilliant and numerous world-wide participation. After Professor Stoilow's estimation (*Oeuvre Mathématique*, Ed. Acad., 1964, p.355), Corneliu Constantinescu "succeeded to extend considerably the hyperbolic measure principle" by introducing the *dispersion* for a three points system and respectively for a set. Thus he precised many classic theorems in value distribution theory (Landau, Schottky, Bermant, Julia) and completed Valiron's results by defining a new class of meromorphic functions with at most one respectively two deficient values.

In 1958 Corneliu Constantinescu defended under professor Stoilow's supervision his doctoral thesis: *On the classification of Riemann surfaces*, published in Acta Mathematica, 102, 1-2 (1959), 47-77. In this dissertation, dedicated to a subject which then represented one of the main research themes in Complex Analysis, he constructed a new boundary F for every Riemann surface R having the unit disk D as universal covering. He used in this view the action of the cover mapping group G over the boundary of D. Let $\mathcal{F}$ be the set of points $\exp(i\theta)$ which are not angular limits of sequences from an orbit Gz, z - an arbitrary point in D; $\mathcal{F}$ is Lebesgue measurable of measure null if R belongs to the class O_G and 2π other-wise, and $F = \mathcal{F}/G$. Further, he established the relation between the boundary F and the Kerékjártó - Stoilow boundary, defined the notion of *indecomposable set* M of F (i.e. whose pre-image $\mathcal{M}$ in F is a measurable set of positive measure but indecomposable as the union of two disjoint, measurable sets of positive measure) and introduced the class U of the Riemann surfaces having at least one indecomposable set on the boundary. Moreover, he gave a new classification principle, by defining the classes $U_{\mathfrak{M}}$ relative to various important families $\mathfrak{M}$ of functions on Riemann surfaces: HB, HD,HD^{α} , $0 \leq \alpha \leq 1$, KB etc; $U = U_{HB}$. These classes $U_{\mathfrak{M}}$ have remarkable properties; in particular with their help C.Constantinescu explained the following paradox: A.Mori (1953) and Z.Kuramochi (1955) proved that the surface obtained from a surface of class $O_{HV} \setminus O_G$, $V = B$ or D , by taking out a disk belongs to the class O_{AV} . However the surfaces from O_{AV} as well as those from O_{HV} have "thin" boundaries while it seemed natural that a surface obtained by extracting a disk should have a "thick" boundary. Corneliu Constantinescu removed this paradox establishing the inclusions: $O_{HV} \setminus O_G \subset U_{HV} \subset O_{AV} \setminus O_G$ and the property of the classes U_{HV} to be invariant by taking out a disk. Thus the concept of "thin boundary" completes its sense: it can include $\mathfrak{M}$ - indecomposable sets. The dissertation also contains many other remarkable results: solution of a problem set by M.Parreau in 1950 , characterization of the indecomposable sets and of the class U by R.S.Martin minimal functions,

connections with Iversen class and with total coverings as well as significant examples of surfaces from classes U_{HV} .

In the meantime Stoilow's Seminar and also the Institute developed by including young mathematicians of great value among which Nicu Boboc, Aurel Cornea, Ciprian Foias, George Gussi, Valentin Poenaru. Thus was possible a very fruitful collaboration between Corneliu Constantinescu and Aurel Cornea ("the C-C tandem" as they were called by professor Miron Nicolescu - the Institute director some years later).

In the frame of this collaboration the two friends realized a deep and systematic analysis of the Martin, Royden and Kuramochi ideal boundaries and generalized for analytic mappings between Riemann surfaces the classic theorems of Fatou-Nevanlinna, Riesz-Lusin-Privalov, Frostman-Nevanlinna, Plessner, Beurling. Moreover, Constantinescu and Cornea introduced a new ideal boundary, called by them the *Wiener boundary* , which plays with respect to the class HB an analogous role as Royden boundary plays relatively to the class HD.

The importance and beauty of these results determined L.V.Ahlfors' proposal that they should write a monograph in the well known collection Ergebnisse der Mathematik und ihrer Grenzgebiete of Springer-Verlag. Thus arised in 1963: *Ideale Ränder Riemannscher Flächen*, representing for Corneliu Constantinescu and Aurel Cornea a new occasion to extend already obtained results and for the specialists in the field a basic book, laying foundations of future researches, among which the monograph series consacrated to Riemann surfaces and at the ends to Riemann manifolds by L.Sario and his collaborators (1966-1980).

Without being able to present even briefly the content in most part original of this reference book, let us underline a thread which runs right through it enlightening the harmonious way in which Wiener boundary equilibrates the theory. Authors classify the four ideal boundaries from two distinct points of view:

1.Boundaries which are simple, reducing to the usual boundary in the case of plane domains, and very convenient in generalizations of classic theorems: Martin and Kuramochi boundaries, and respectively, complicated boundaries but efficient in proofs: Wiener and Royden boundaries.

2. Boundaries adequate to classes B : Martin and Wiener, and boundaries specific to classes D : Kuramochi and Royden.

Let us also mention the essential place dedicated in the book to superharmonic functions, potential theory and Dirichlet problem, which are dealt with in a personal systematization, at different stages of the exposure and finally in the frame of Kuramochi compactification. As a matter of fact potential theory constituted the subject of Stoilow's Seminar in the scholar year 1959-60, and the lectures presented then together with N.Boboc appeared as a monograph in Academy Publishing House (1962).

In 1963 the "Gh.Lazar" prize of the Romanian Academy is awarded to Corneliu Constantinescu.

Beginning with 1962, C.Constantinescu, A.Cornea and N.Boboc oriented them-selves to the general theory of the potential and between 1963-66 they organized in co-operation with Ioan Cuculescu a seminar on the connections with the probability theory. To this prestigious team joined new young mathematicians: Gh.Bucur, C.Meghea, I.Ichim, G.Isac (in that period C.Meghea, I.Ichim and I.Isac were writing their doctoral thesis under Constantinescu's supervision) and others, which gradually will form the present Romanian potential school.

However, coming back to the sixties, C. Constantinescu alone, together with A.Cornea or with A.Cornea and N.Boboc, published another cycle of fundamental papers in the axiomatic of potential theory, i.e. according to the present terminology in harmonic spaces theory. They began by improving Brelot's and Brauer's axiomatizations gradually rebuilding the entire theory: solution of Dirichlet problem for arbitrary open sets of harmonic spaces with extension to the nonlinear case, construction of balayage theory for functions and measures, generalization of Wiener boundary theory to harmonic spaces of Brelot's type, study of the specific order on the convex cone of superharmonic functions, construction and study of Markov processes associated to harmonic spaces, equicontinuity, kernels and nuclearity on harmonic spaces. (We followed in this succinct enumeration A.Cornea's synthesis in the homage volume: *The Institute of Mathematics of the Academy, 20 years of activity 1949 - 1969*, Ed. Acad., 1970, p.66.)

These papers achieved a period of about 20 intense research years in potential theory all over the world, which claimed for a general account. These were the conditions under which C.Constantinescu and A.Cornea wrote the monograph: *Potentialtheory on harmonic spaces*, published in the celebrated collection Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen of Springer-Verlag. In the volume preface H.Bauer, one of the pioneers of this field, wrote: "C.Constantinescu and A.Cornea, who both had considerably influenced the development of the theory, undertook the difficult task to present our present-day knowledge of harmonic spaces as completely as possibleThe monograph will have a strong impact on the furture research in the field of potential theory."

Around 1970 the contacts of Romanian mathematics with the exterior intensified. Important scientific conferences were held in our country to which participated great mathematicians as L.V.Ahlfors, H.Cartan, G.Choquet, J.Dieudonné, F.Hirzebruch, B.V.Gnedenko, A.Grothendieck, B.Sz.-Nagy, S.Sobolev and others. In this atmosphere of certain political relaxation (1972), C. Constantinescu obtained the approuval to visit some western universities where he was repeatedly invited. Recollections of the danger and injustices from his youth, the incertitude for the future of the fundamental research by us, maybe other reasons too, his mother was of German origin, determined him to remain abroad.

His creation way was directed now to wider fields of mathematics connected also to the lectures given as an invited professor at universities in Switzerland (École Polytechnique Fédérale de Lausanne (1972-73), ETHZ - Eigenössische Technische Hochschule Zürich, (1973-76)), as a full professor in Germany (Technische Universität Hannover (1976-78)) and finally again in Switzerland at ETHZ, where he became a full professor by a large competion (1978-96).

Even in Romania he was deeply interested and published papers or books on other mathematical fields: Set Theory, Theory of Formal Systems, Topology, Analysis on Manifolds, Real Analysis. Now, in parallel with numerous original research papers concerning especially Measure and Integration Theory, he wrote many textbooks from Analysis to Linear Algebra and Topological Spaces. At the same time he concluded again every research period through a synthesis monograph. Thus appeared: *Duality in measure Theory*, L.N. in Math.796, Springer-Verlag (1980), *Spaces of measures*, W. de Gruyter, Studies in Math. 4 (1984), *Integration Theory - Measure and Integral*, J.Willey & Sons (1985) and *Advanced Integration Theory*, Kluwer Acad. Publ. (1998), the two last works published together with his former doctoral students K.Weber, and W.Filter and K.Weber respectively, in collaboration with Alexia Sontag. Since 1989, he is working in C^*-algebra theory.

To present even concisely this period so rich and extensive in topics,a group of specialists would be necessary. We will shortly insist on some fragments from introductions or prefaces of these books, since they reveal specific features of Corneliu Constantinescu's spirit and of his work, which brought order after a stage of strong but chaotic development of certain mathematical fields.

Thus the Introduction to the impressive *Spaces of Measures* points out as the starting point for the book the explosion in the seventies of generalizations in measure theory. This determined Corneliu Constantinescu to analyse the huge material accumulated, to collect the common ideas and to present the theory "in a unified manner". He chose the natural frame of the function spaces and constructed a general theory whose results could be applied in every important particular case: spaces of supersummable families, spaces of measures, spaces of exhaustive additive mappings, but also some spaces of group homomorphisms and duals of M-spaces. In this book, as well as in the whole work, precision and systematization are dominant. Let us give a significant example. In order to avoid the difficulties due to the abstract and general character of the first chapters he imagined an ingenious way to use references (which indicate for each paper the place in the book where it is quoted) and historical remarks, so that the sense and content of notions and arguments be clarified.

A similar procedure lead also to the achievement of the treatise *Advanced Integration Theory*. Fascinated by the unification problem of the two trends in integration theory: the abstract and the topological one, Authors succeeded to solve the problem by using new definitions in the abstract theory. They chose the vast frame of vector lattices and defined a notion of integral, which is the best possible for the proposed aim. The presentation observes the classic format: "definition - theorem - proof ", however this does not exclude the narration and commentary, bringing out in full relief ideas of proofs and history of notions, which gives life to the text.

However from this so abstract world, Corneliu Constantinescu returns to the Riemann surface topology in the context of Teichmüller spaces, rendering homage together with Dan Burghelea to the memory of the loved professor Simion Stoilow in the volume *Analysis and Topology*, World Scientific (1998). There they publish the paper: *Cutting and gluing back along a closed simple curve on a Riemann surface* (pp. 191-213), in which they study the analytic structure change of the surface under these operations by means of quadratic differentials, Kodaira-Spencer theory and Serre duality.

The love for his natal country is constantly reflected in all these years of exile but also of international affirmation, from that touching dedication of the volume *Mass und Integral* (Autographie zur Vorlesung, ETHZ, Verlag der Fachvereine, Zürich, 2. Auflage, 1990) to professor Miron Nicolescu: "*Mein erster Kontakt mit der modernen Integrations- theorie hat in der schönen Vorlesung von Professor Miron Nicolescu stattgefunden. Aus Dankbarkeit sei ihm dieses Skriptum gewidmet*", to the name "*mioritic space*" given by him to one of the important concepts he defined in *Spaces of Measures*, to the generous book donation he obtained in 1990 for the library of the Institute of Mathematics and to the invaluable support and friendship devoted to Romanian colleagues.

Monographies

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