LIBERTAS MATHEMATICA, vol XXVIII (2008)

A Physical Approach to Goldbach's Conjecture and Fermat's Last Theorem

CLASSROOM NOTE

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Abstract. In this work, we introduce and specify how to apply physics to solve or estimate some problems in number theory and analysis. More precisely, we use center of mass (centroid) to show how to find or estimate sum of positive numbers, and definite integrals. Consequently, as a corollary, we write an alternative expression of Fermat's Last Theorem for positive integers and predict some impossible cases. Finally, we express an alternative method via center of mass toward a solution (discussion) of Goldbach's Conjecture.

Keywords: k-Mass-System of n points, center of mass (centroid).

1. Applying Physics to Mathematics

We usually in physics apply mathematics to solve or express a physical phenomenon by finding a mathematical model (expression). In this work, our goal is to show the capability of physics for solving (approximating) mathematical problems (expressions). Also, author assumes that the reader is familiar to the concept of center of mass or centroid; and the problems in number theory that we use in this article. For a detailed study of center of mass, the reader is referred to [1] and [4]; and for the related problems in number theory, see [2] and [3]. Next, in order to justify the concept of this subject, we start with two simple examples from number theory as follows.

Example 1. *What is the sum $S = 1 + 2 + \cdots + n$? Suppose each number is a physical point having one unit of mass and $x_i = i$ is the distance of the ith point (number) from the origin for each $i = 1, 2, \ldots, n$. Hence, by the definition of centroid, $S = \sum_1^n m_i x_i = c \sum_1^n m_i$ implies $S = cn$, where $c = 1/2(n+1)$ is the center of mass of the n points on x-axis which is precisely in the middle of the interval $[1, n]$ since all the points in the mass system have equal masses and distributed uniformly; and $m_i = 1$ unit of mass for each $i = 1, 2, \ldots, n$.*

Example 2. *Let $x_1, x_2, \ldots, x_n$ be a strictly increasing finite sequence of positive integers. What is $S = \sum_1^n x_i$? Again, we assume each number x_i is a physical point with mass $m_i = 1$ for each $i = 1, 2, \ldots, n$. Now, we apply the above centroid method and get $S = cn$, where c is the center of mass of the system.*

Remark 1. *Note that in a mass system of points, the center of mass or equilibrium point of the system is closer to the heavier (massive) part of the system. Hence, any method or means that helps us to a closer approximation value of the centroid yields a better approximation of the unknown in the mathematical expression.*

Definition 1. *Let $n \geq 2$ and $k \geq 1$ be positive fixed integers. A k-mass-system of n points, denoted by $M(k,n)$, is a strictly increasing sequence $x_1 < x_2 < \cdots < x_n$ of n positive points on x-axis with each point x_i having $m_i = x_i^k - 1$ unit(s) of mass for each $i = 1, 2, \dots, n$.*

Theorem 1. *Let $M(k,n)$ be a k-mass-system for fixed integers $n \geq 2$ and $k \geq 1$. Let $S_k = \sum_{i=1}^n x_i^k$. Suppose each number x_i is a physical point having $m_i = x_i^{k-1}$ unit(s) of mass. Also, assume $S_0 = n$. Then $S_k = nc_1c_2\cdots c_k$, where c_i is the center of mass of the i-mass-system for each fixed $i = 1, 2, \dots, k$.*

Proof. From the definition, it follows that $\sum_{i=1}^n m_i x_i / \sum_{i=1}^n m_i = c_k$ is the center of mass of n points (x_i's on x-axis) with m_i unit(s) of mass for each $i = 1, 2, \dots, n$. Hence, $S_k = \sum_{i=1}^n m_i x_i = c_k S_{k-1}$, where c_k is the center of mass of the mass system $M(k,n)$ with n points $x_1 < x_2 < \cdots < x_n$ on x-axis. Also, note that $S_0 = n$ by assumption. Now, from the above, it is clear that $S_k/S_{k-1}S_{k-1}/S_{k-2}\cdots S_1/S_0 = c_k c_{k-1}\cdots c_1$. Thus, $S_k = nc_1c_2\cdots c_k$. □

Corollary 2. *Suppose $x < Y$ is a mass system of 2 positive fixed integers (points) on x-axis. Then for any fixed integer $k \geq 3$, the statement "$x^k + y^k = S_k = 2c_1c_2\cdots c_k$ can not be kth power of an integer" is equivalent to Fermat's Last Theorem for positive integers.*

Remark 2. *In the above corollary, suppose $S_k = Z^k$ for some integer z and positive integer $k \geq 3$. Thus, if the product of c_i's (the center of masses) is an integer, then z can not be an odd integer. Furthermore, If $c_1c_2\cdots c_k$ is a fraction with a denominator different from 1 and 2, then the equality $S_k = z^k$ is never valid for any integer z. Actually, $c_1c_2\cdots c_k$ must always be an integer or a fraction (of course, in reduced form) with denominator 2 since S_k is always a positive integer. Now, let $x < y$ be two fixed positive integers, and $c = Max\{c_1, c_2, \dots, c_k\}$. Then $S_k = x^k + y^k = z^k = 2c_1c_2\cdots c_k \leq 2c^k < 2y^k$ implies $z < 2^{1/k}y$. Therefore, for each fixed positive integer y, there always exists a sufficiently large integer k_0 such that $0 \leq 2^{1/k}y - y < 1$ for all $k \geq k_0$. Hence, for sufficiently large k, y is the largest integer less than $2^{1/k}y$. Consequently, $z \leq y$, which is impossible since x is a positive integer. From this, we can conclude that for any fixed positive integers $x < y$, there always exists a sufficiently large positive integer k_0 such that $x^k + y^k = z^k$ is impossible for all positive integers z and all $k \geq k_0$.*

Next, we extend the above theorem to a class of definite integrals as follows.

Theorem 3. *Suppose k is a fixed positive integer and $a < b$ are two real numbers. Let for each $i = 1, 2, \dots, k$, $S_i = \int_a^b x x^{i-1} f(x)dx \neq 0$, where x^{i-1} is the density function of a plate bounded by $f(x)$, x-axis, $x = a$, and $x = b$. Also, assume that $S_0 = \int_a^b f(x)dx$ is the area under the curve (area of the plate). Then $S_k = S_0c_1c_2\cdots c_k$, where c_i is the x-coordinate of the center of mass of the plate for each $i = 1, 2, \dots, k$.*

Proof. The proof is similar to the argument in the above theorem. Note that $c_i = S_i/S_{i-1}$ is the x-coordinate of the center of mass of the plate for each $i = 1, 2, \dots, k$. □

2. Goldbach's Conjecture

Goldbach's conjecture is one of the oldest unsolved problems in number theory. It states that for every even integer $n > 2$, there exist two prime numbers p and q such that $n = p+q$. Suppose each of p and q has one unit of mass. Thus $c = (p+q)/2$ is the centroid of the system, which implies $p + q = 2c$. Thus, if we assume that the conjecture is valid, then $n = 2c$. now, let $n > 2$ be an even integer. In this case, $n/2$ is the center of mass of the system of two primes with each having one unit of mass, which are at equal distance from their center of mass $c = n/2$. Next, we extend this method for two prime numbers $p < q$ with different masses. Suppose $n = p + q$ and $p < q$ are two primes with positive real-valued masses s and t $(s < t)$, respectively. In this case, the centroid of the system $c = (sp + tq)/(s + t)$ locates closer to q which is heavier than p. Thus, $n/2 \leq c \leq q$. Simillarly, if we assume $s > t$, then $p \leq c = (sp+tq)/(s+t) \leq n/2$ whenever we assume p has s units of mass and q has t units of mass.

Remark 3. *In the above discussion on Goldbach's Conjecture, it is clear that from a philosophical point of view, the direction of consideration and argument changes from a pure number-theoretic to a somewhat analysis type which could be helpful on study of the conjecture. Also, for those who are interested to test and challenge the conjecture via a computer programming language, or heuristic argument, choices are broader to test.*

Finally, we close this article with a general overview on the basic concepts of the subject as follows.

3. A General Overview

Note that the basic concept underlying this idea is that for any mathematical expression with k elements or terms that models a phenomenon of physics, (life science, social sciences) we can find or estimate (measure) one term whenever we get the other $k - 1$ terms of the expression. From this, we can establish a new branch of science (human thought) and call it *Physical Mathematics.* This method might be very useful to calculate or estimate the value of difficult definite integrals (with no analytic solutions at hand) vvia the measurement of the centroid and/or (area, volume) of an object (physical shape) and integration of simpler integrands. Thus, we have to use and select a simple way to calculate (estimate) the centroid (area, volume) practically (experimentally) or via a computer simulation such as Monte Carlo Method, or any other efficient and available practice. Actually, according to my knowledge, there is no counterpart to this idea in the literature, and possibly there might be some common points with the experimental mathematics.

References

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