NUMERICAL ALGORITHM REGARDING SYMMETRIC DISCRETE POLLING SYSTEM

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Abstract: Discrete polling systems find a wide application in the different areas, such as transportation, computer systems, telecommunications, production, and others [1]. In this article it is studied symmetric (uniform) discrete polling systems with nonzero server switchover time for three types of service. It is proposed a numerical algorithm for the mean waiting time, according to type of service. The numerical examples of modeling mean waiting time are presented.
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1. Introduction

Polling system is a multi-queue system with a single server which visits the queues according to a polling order and serves the customers from these queues. Furthermore, polling models are applicable in situations in which several users compete for access to a common resource which is available to only one at a time, such as traffic and transportation systems and production systems.

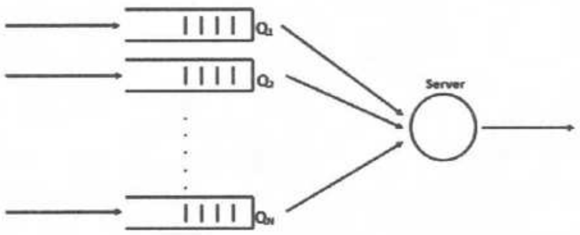

Figure 1. Discrete polling system

But in majority of the works devoted to the analysis of polling systems it is considered that the switching of service process from one to another user occurs at once, which does not reflect the real situation. In real systems to make this switching, it is lost some time, which as rule is random. For example, models that take into consideration these losses are polling models with continuous time and exhaustive service with semi-Markov delays, which were investigated in some recently publications (see for example [2-5]) and studied symmetric polling discrete time systems. One of the important performance measures of the discrete polling system is the mean waiting time of the customers at the stations. Below it is proposed a numerical algorithm for calculation the mean waiting time for polling model with discrete time and nonzero server switchover times for different types of service, such as exhaustive, gated and 1-limited services discipline. The algorithm performs the analytical expressions obtained in the form of generating functions in [6]. The numerical examples and results of modeling are presented below.

2. Notation and Preliminaries

Discrete polling system with nonzero switchover times consists of one server and N queues Q_i, $i=\overline{1,N}$. Time is slotted into equal-length intervals $\{[t,t+1), t=0,1,2,...\}$.

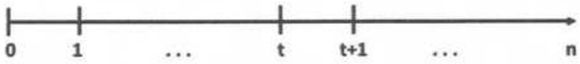

Figure 2. Time axis

The input flow of messages represent a set of independent random variables $\{X_i(t), t=0,1,2,...\}$ where $X_i(t)$ is the number of messages that arrive at the queue Q_i, $i=\overline{1,N}$ in the interval $[t,t+1)$. The random variables $X_i(t), t\geq 0, i=\overline{1,N}$ are identically distributed. The expectation and variance of these variables are μ_i and $\sigma_i^2, i=\overline{1,N}$, respectively.

By the random variable $S_i(t), t=0,1,...$ is denoted the duration of switching from queue Q_i to queue Q_{i+1} in the interval $[t,t+1)$. These random variables $S_i(t), t=0,1,...$ are independent and identically distributed with the expectation r_i and variance $\delta_i^2, i=\overline{1,N}$.

3. Mean waiting time for symmetric discrete polling system

In particular case, for symmetric polling system $X_i(t)=X(t), i=\overline{1,N}$, so the expectation $\mu_i=\mu$ and variance $\sigma_i^2=\sigma^2$.

We consider that $S_i(t)$ is given by Bernoulli distribution: $P\{S_i(t)=1\}=p$, $P\{S_i(t)=0\}=1-p$.

Therefore $r_i=r=p$ and $\delta_i^2=\delta^2=p(1-p), i=\overline{1,N}$.

For the symmetric system, the mean waiting time is as follows [6]:

$$M=\frac{1}{2}\left(\frac{\delta^2}{r}+\frac{\sigma^2}{\mu(1-N\mu)}+\frac{Nr(1-\mu)}{1-N\mu}\right).$$

For this type of system there are various options for the service disciplines at the queue. The difference consists in the number of messages which may be served in a queue during a server's visit to that queue. Among the service disciplines we analyzed:

1. *exhaustive service discipline*, when the server serves all messages until the queue is empty;
2. *gated service discipline*, when the server serves those messages that had arrived in the queue before its arrival at Q_i;
3. *1-limited service discipline*, when the server serves only one message from each queue.

For the symmetric queueing system according to the mentioned above type of services, we have the following formulas [6]:

a) for the exhaustive service

$$M1 = \frac{1}{2}\left(1 + \frac{\sigma^2}{\mu(1-N\mu)}\right), \tag{1}$$

b) for the gated service

$$M2 = \frac{1}{2}\left(\frac{\delta^2}{r} + \frac{\sigma^2}{\mu(1-N\mu)} + \frac{Nr(1+\mu)}{1-N\mu}\right), \tag{2}$$

c) for the *1*-limited service

$$M3 = \frac{1}{2}\left(\frac{\delta^2}{r} + \frac{\sigma^2(1+Nr)}{\mu(1-(1+r)N\mu)} + \frac{N\delta^2\mu}{1-N\mu-Nr\mu}\right). \tag{3}$$

4. Numerical algorithm

The presented algorithm solves the formulas (1)-(3).

Input: N, n, P, $\{x_i\}_{i=1}^{n}$.

Output: μ, σ^2, r, δ^2, $M1$, $M2$, $M3$.

Description:

1. The expectation (μ) for the number of messages which arrives in n slots, $n = 1, 2, \ldots$ is calculated;
2. The variance (σ^2) for the number of messages which arrives in n slots, $n = 1, 2, \ldots$ is calculated;
3. The variance (δ^2) for the switchover time from queue Q_i to Q_{i+1}, $i = \overline{1, N}$ is calculated;
4. The mean waiting times is calculated for the symmetric system which depends on:

- the exhaustive service $(M1)$, according to the formula (1);
- the gated service $(M2)$, according to the formula (2);
- the 1-limited service $(M3)$, according to the formula (3).

5. Numerical examples

Example 1. Consider a symmetric queueing system of discrete polling type with cyclic order and nonzero server switchover times formed by one server and $N = 10$ queues. The number of messages from each slot $n = \overline{1, 8}$ is given by $x_i = \{1, 5, 3, 1, 4, 2, 3, 1\}$. The probability that server will connect from a queue to another for service is equal to $P = 0.0112$.

Table 1. Mean waiting times for the described above system and type of services

$X(t)$	$\mu = 2.500000$
	$\sigma^2 = 2.285714$
$S(t)$	$r = 0.011200$
	$\delta^2 = 0.011075$
$M1$	0.499909
$M2$	0.467432
$M3$	0.467995

Example 2. Let be a symmetric queueing system of discrete polling type with cyclic order and nonzero server switchover times. It is formed by one server and $N = 10$ queues. The number of messages from each slot $n = \overline{1, 15}$ is given by $x_i = \{1, 5, 3, 4, 7, 6, 3, 1, 5, 3, 5, 2, 1, 7, 2\}$. The probability that server will connect from a queue to another for service is equal to $P = 0.324$.

Table 2. Mean waiting times for the described above system and type of services

$X(t)$	$\mu = 3.666667$
	$\sigma^2 = 4.380952$
$S(t)$	$r = 0.324000$
	$\delta^2 = 0.219023$
$M1$	0.499163
$M2$	0.109288
$M3$	0.200274

Example 3. Let be a symmetric queueing system of discrete polling type with cyclic order and nonzero server switchover times. It is formed by one server and $N = 20$ queues. The number of messages from each slot $n = \overline{1, 14}$ is given by $x_i = \{5, 3, 2, 4, 1, 3, 6, 7, 8, 2, 1, 3, 1, 3\}$. The probability that server will connect from a queue to another for service is equal to $P = 0.208420$.

Table 3. Mean waiting times for the described above system and type of services

$X(t)$	$\mu = 3.500000$
	$\sigma^2 = 5.038462$
$S(t)$	$r = 0.208420$
	$\delta^2 = 0.164981$
$M1$	0.499658
$M2$	0.249432
$M3$	0.282206

Example 4. Let be a symmetric queueing system of discrete polling type with cyclic order and nonzero server switchover times. It is formed by one server and $N = 18$ queues. The number of messages from each slot $n = \overline{1, 16}$ is given by $x_i = \{2, 3, 4, 6, 2, 1, 7, 6, 4, 3, 2, 1, 5, 6, 4, 3\}$. The probability that server will connect from a queue to another for service is equal to $P = 0.402$.

Table 4. Mean waiting times for the described above system and type of services

$X(t)$	$\mu = 3.687500$
	$\sigma^2 = 3.562500$
$S(t)$	$r = 0.402000$
	$\delta^2 = 0.240396$
$M1$	0.499501
$M2$	0.032194
$M3$	0.169119

6. Conclusion

From the theoretical-fundamental point of view the proposed algorithm is simple and doesn't present the difficulties to compute it in some programming languages. But, from applicative point of view the presented numerical model can be used to establish particular minimal values for the mean waiting time depending on the input values of Bernoulli distribution. Therefore, from presented examples in Tables (1)-(4), it is observed that minimal value of the mean waiting time is achieved for $S(t)$ with the parameters $r = 0.402000$ and $\delta^2 = 0.240396$. We conclude that this studied model helps up to find particular cases for minimizing the mean waiting time.

7. References

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