INFLUENCE OF START VALUES OVER THE GLOBAL OPTIMIZED SHAPE OF FLYING CONFIGURATIONS, IN SUPERSONIC FLOW

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Abstract: The global optimization (GO) of the shape of a flying configuration (FC) (namely, the simultaneous optimization of its camber, twist and thickness distributions and also of the similarity parameters of its planform), in order to reach a minimum drag at cruising Mach number, leads to an enlarged variational problem with free boundaries. An own optimum-optimorum theory was developed in order to solve this enlarged variational problem with free boundaries.The dependence of the GO shape on the change one by one of start values of optimization, it is the chosen cruising Mach number, the lift and the pitching moment coefficients, is here analyzed.

1. Introduction

In some previous papers, the author has developed three-dimensional analytical hyperbolic potential solutions for the supersonic flow over FCs. These discontinuous solutions use minimal singularities located only along the singular lines (like subsonic leading edges, junction lines wing/fuselage, wing/leading edge flaps, etc.), fulfill the jumps along these singular lines, are balanced and are easy to be applied, because they are written in integrated forms, as in [1-3]. These analytical solutions were used as start solutions for the determination of the inviscid GO shape of FC. The classical optimization problem consists in the determination of the shape of the surface of an elitary FC with given planform, which leads to a variational problem with fixed boundaries. The determination of the GO shape of FC (namely, the simultaneous optimazation of its camber, twist and thickness distributions and *also* of the similarity parameters of its planform), leads to an enlarged variational problem with free boundaries. An own optimum-optimorum (OO) theory was developed in order to solve this enlarged variational problem. According to this OO theory, a lower limit hypersurface of the drag coefficients of elitary FCs versus the corresponding set of similarity parameters of planform is defined. The elitary FCs corresponding to the optimal value of the optimal set of similarity parameters, which is obtained by the numerical determination of the position of the minimum of the lower limit hypersurface is, at the same time, the GO shape of the FC belonging to the given class of elitary FCs.

The dependences of the GO shape of FC on the change of the start values of optimization one by one, namely the given values of the cruising Mach number M_∞, of the lift and pitching moment coefficients are here analyzed.

2. Computation of Hyperbolic Potential Supersonic Flow Over a Flying Configuration

Let us further consider a delta wing with arbitrary camber, twist and thickness distributions, which is flying at the cruising Mach number M_∞. Dimensionless coordinates are used for the computation of the distributions of velocity's components:

$$\tilde{x}_1 = \frac{x_1}{h_1} \ , \ \tilde{x}_2 = \frac{x_2}{\ell_1} \ , \ \tilde{x}_3 = \frac{x_3}{h_1} \ . \ (\ \tilde{y} = \frac{\tilde{x}_2}{\tilde{x}_1}) \quad (1)$$

The downwashes w and w^* are supposed to be expressed in form of superposition of homogeneous polynomes with arbitrary coefficients, namely:

$$w \equiv \tilde{w} = \sum_{m=1}^{N} \tilde{x}_1^{m-1} \sum_{k=0}^{m-1} \tilde{w}_{m-k-1,k} \left|\tilde{y}\right|^k \quad ,$$

$$w^* \equiv \tilde{w}^* = \sum_{m=1}^{N} \tilde{x}_1^{m-1} \sum_{k=0}^{m-1} \tilde{w}^*_{m-k-1,k} \left|\tilde{y}\right|^k \quad . \qquad (2a,b)$$

For a given equation of FC's surface, the coefficients of the downwashes and the similarity parameter $\nu = B\ \ell$ ($\ell = \ell_1 / h_1$, $B = \sqrt{M_\infty^2 - 1}$), of the planform are known and ℓ, ℓ_1, h_1 are the dimensionless span, the half-span and the depth of the planform of the delta wing.

If the principle of minimal singularities (which fulfill the jumps of the velocity's components) and the hydrodynamical analogy of Carafoli are used, the following expressions for the axial disturbances of the thin and of the thick-symmetrical components of the FC with subsonic leading edges are obtained:

$$u = \ell \sum_{n=1}^{N} \tilde{x}_1^{n-1} \left\{ \sum_{q=0}^{E\left(\frac{n}{2}\right)} \frac{\tilde{A}_{n,2q} \tilde{y}^{2q}}{\sqrt{1-\tilde{y}^2}} + \sum_{q=1}^{E\left(\frac{n-1}{2}\right)} \tilde{C}_{n,2q} \tilde{y}^{2q} \cosh^{-1} \sqrt{\frac{1}{\tilde{y}^2}} \right\} \ ,$$

$$u^* = \ell \sum_{n=1}^{N} \widetilde{x}_1^{n-1} \left[\sum_{q=0}^{E\left(\frac{n-2}{2}\right)} \widetilde{D}^*_{n,2q} \widetilde{y}^{2q} \sqrt{1-\nu^2\widetilde{y}^2} \right.$$

$$+ \sum_{q=0}^{n-1} \widetilde{\widetilde{H}}^*_{nq} \, \widetilde{y}^q \left(\cosh^{-1} M_1 + (-1)^q \cosh^{-1} M_2 \right)$$

$$\left. + \sum_{q=1}^{E\left(\frac{n-1}{2}\right)} \widetilde{C}^*_{n,2q} \widetilde{y}^{2q} \cosh^{-1} \sqrt{\frac{1}{\nu^2 \widetilde{y}^2}} \; \right] . \qquad (3a,b)$$

$$\left(M_{1,2} = \sqrt{\frac{(1+\nu)(1 \mp \nu \, \widetilde{y})}{2\nu(1 \mp \widetilde{y})}} \right)$$

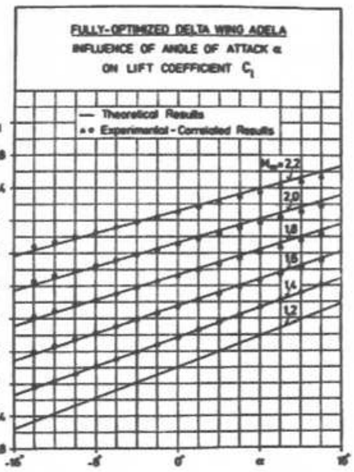
The lift and the pitching moment coefficients of the delta wing, computed with the hyperbolic potential theory, are the following :

$$C_\ell \equiv 8\ell \int_{\widetilde{O}\widetilde{C}_1\widetilde{A}_1} \widetilde{u} \, \widetilde{x}_1 d\widetilde{x}_1 dy \quad ,$$

$$C_m \equiv -8\ell \int_{\widetilde{O}\widetilde{C}_1\widetilde{A}_1} \widetilde{u} \, \widetilde{x}_1^2 d\widetilde{x}_1 dy \quad . \qquad (4a,b)$$

The lift, pitching moment and the pressure coefficients of the upper side of six delta FC's models (namely the wedged delta, the double wedged delta, the delta wing fitted with central conical fuselage, the global optimized delta wing Adela, the global optimized and fully-integrated wing-fuselage models Fadet I and Fadet II) were measured in the trisonic wind tunnel of DLR-Köln, in the frame of research projects of the author, sponsored by the DFG. The comparison of theoretical and experimental-correlated values of the lift and pitching moment coefficients of the global optimized delta wing model Adela , given in (Fig. 1), are presented in the (Fig. 2a,b).
These comparisons show the very good agreements between experimental and theoretical analytical hyperbolical potential solutions for the lift and pitching moment coefficients of FCs with subsonic leading edges, at moderate angles of attack (deduced by the author in closed forms, as in [1-3]), lead to the following remarks.
- The validity of the three-dimensional hyperbolic analytical potential solutions for the axial disturbance velocity with the chosen balanced minimal singularities and the corresponding developed software for the above aerodynamic coefficients are confirmed;
- the influence of friction upon these coefficients is neglectable;
- the flow is laminar, as supposed here and it remains attached in supersonic flow, for larger angles of attack than by subsonic flow;
- if the FC is flattened enough, has sharp leading edges and flies at moderate angles of attack, the flight with characteristic surface, instead of the flight with shock wave surface, is confirmed.

Fig. 1 The Global Optimized Delta Wing Model Adela



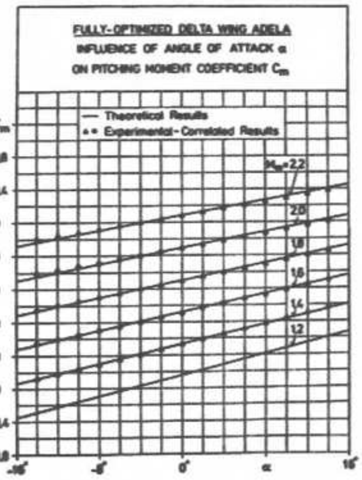


Fig. 2a,b The Lift and the Pitching Moment Coefficients of the Global Optimized Delta Wing Model Adela

The inviscid drag coefficients of the thin and thick-symmetrical wing components and of the delta wing model are:

$$C_d \equiv \ell \widetilde{C}_d = 8\ell \int_{\widetilde{O}\widetilde{A}_1\widetilde{C}} \widetilde{u}\, \widetilde{w}\, \widetilde{x}_1 d\widetilde{x}_1 d\widetilde{y} \quad ,$$

$$C_d^* \equiv \ell \widetilde{C}_d^* = 8\ell \int_{\widetilde{O}\widetilde{A}_1\widetilde{C}} \widetilde{u}^* \widetilde{w}^* \widetilde{x}_1 d\widetilde{x}_1 d\widetilde{y} \quad ,$$

$$C_d^{(i)} \equiv \ell \widetilde{C}_d^{(i)} = \ell \left(\widetilde{C}_d + \widetilde{C}_d^* \right) \quad . \qquad \text{(5a-c)}$$

For the computation of the total drag of the GO shape of FC own developed hybrid solutions for the full partial-differential equations (PDEs) of the three-dimensional, compressible Navier-Stokes layer (NSL), are here proposed, as in [1-4]. The hyperbolic potential solutions are replaced with hybrid NSL's solutions, which use the potential solutions of the same FC twice, namely: at the edge of the NSL and to reinforce the velocity's components. A spectral coordinate η is here introduced, it is:

$$\eta = \frac{x_3 - Z(x_1,\ x_2)}{\delta(x_1,\ x_2)} \quad . \qquad (0 \le \eta \le 1) \qquad (6)$$

The velocity's components are expressed as products between the potential solutions with polynomes with free coefficients:

$$u_\delta = u_e \sum_{i=1}^{N} u_i\, \eta^i \ , \qquad v_\delta = v_e \sum_{i=1}^{N} v_i\, \eta^i ,$$

$$w_\delta = w_e \sum_{i=1}^{N} w_i\, \eta^i . \qquad \text{(7a-c)}$$

The proposed forms for the here introduced logarithmic density function $R = \ln \rho$ and, for the absolute temperature T, are the following:

$$R = R_w + \left(R_e - R_w \right) \sum_{i=1}^{N} r_i\, \eta^i \quad ,$$

$$T = T_w + \left(T_e - T_w \right) \sum_{i=1}^{N} t_i\, \eta^i \quad . \qquad \text{(8a,b)}$$

The pressure p is computed by using the physical equation of perfect gas and, for the viscosity μ, an exponential law is used:

$$p = R_g\, \rho\, T = R_g\, e^R\, T \ , \quad \mu = \mu_\infty \left(\frac{T}{T_\infty} \right)^{n_1} . \qquad \text{(9a,b)}$$

The free coefficients u_i, v_i, w_i, r_i and t_i are used to satisfy the NSL's PDEs in some chosen points. The coefficients r_i are used to satisfy the continuity equation and are determined only as functions of the coefficients of the velocity's components, by solving a linear algebraic system and the coefficients t_i satisfy the PDE of absolute temperature and are also obtained only as functions of the coefficients of the velocity's components by solving of a transcendental algebraic system. The coefficients of velocity's components are determined by using the impulse PDEs, which are iteratively solved.

The hybrid solutions of the NSL's PDEs are useful for the computation of the friction drag coefficient.

The skin friction coefficient at the wall takes the form:

$$\tau_{x_1}^{(w)} \equiv \tau_{x_1} \Big|_{\eta=0} = \mu_f \frac{\partial u_\delta}{\partial \eta} \Big|_{\eta=0} = \mu_f u_1 u_e \ .$$

The friction and total drag coefficients of the delta wing with arbitrary camber, twist and thickness distributions are:

$$C_d^{(f)} = 8\ \nu_f u_1 \int_{\widetilde{O}\widetilde{A}_1\widetilde{C}} u_e \widetilde{x}_1\, d\widetilde{x}_1\, d\widetilde{y} \quad ,$$

$$C_d^{(t)} = C_d^{(f)} + C_d^{(i)} \quad . \qquad \text{(10a,b)}$$

The hybrid numerical solutions of the NSL's PDEs have important analytical properties like: correct last behaviors, correct and balanced jumps along the singular lines , the condition of the characteristic surface is automatically fulfilled, is matched with the outer hyperbolic potential flow, fulfills the non-slip conditions on the surface of FC and satisfies the NSL's PDEs in a chosen number of points.

3. Influence of Variation of Start Values of the Optimization on the GO shape of FC

Let us consider two GO shapes of FCs with the same area of their planforms and with the same constraints, namely the GO shapes of the fully-integrated models Fadet I and Fadet II, which are of minimum drag, at two different cruising Mach numbers $M_\infty = 2.2$ and, respectively, $M_\infty = 3$.

If the cruising Mach number increases, the maximal depth, of the planform and the camber and twist of the GO surface of FC increase, and its span decreases.

The multipoint design of GO shape of FC at two different cruising Mach numbers can be realized by morphing, by using movable leading edge flaps.

Two enlarged variational problems with free boundaries occur. The first one consists in the determination of the GO shape of the FC with flaps in retracted position, which represents the unmovable part of the entire FC, is of minimum drag at higher cruising Mach number and the second one consists in the determination of the GO shape of the flaps in such a manner that the entire FC, with flaps in stretched position, is of minimum drag, at the second, lower cruising Mach number.

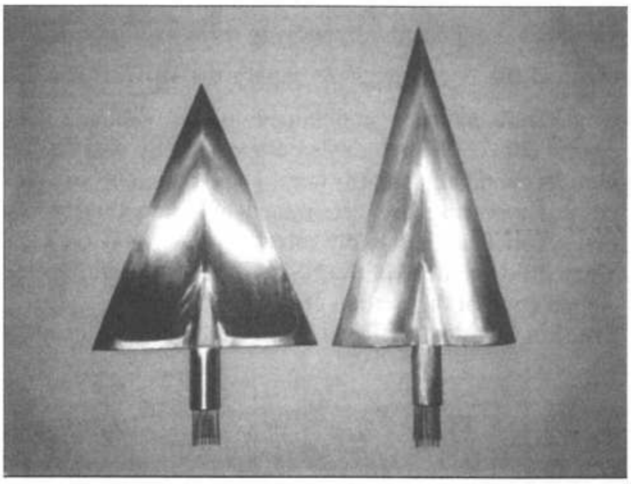


Fig. 3a,b The Global Optimized and Fully-Integrated Models Fadet I and Fadet II of Minimum Drag, at Cruising Mach Numbers 2.2 and 3.

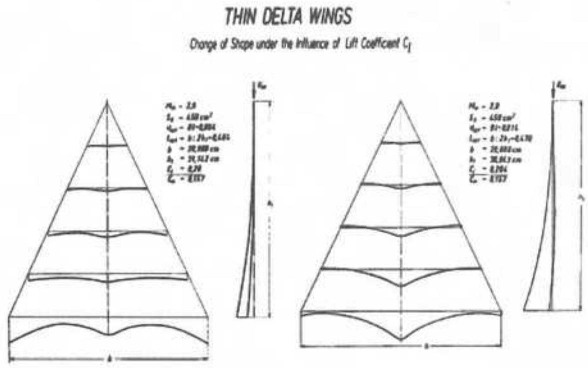


Fig. 4a,b Influence of Increasing Lift Coefficient over the GO Shape of the Thin Delta Wing.

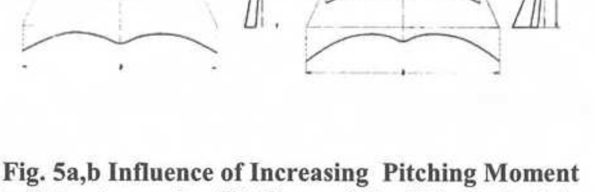


Fig. 5a,b Influence of Increasing Pitching Moment Coefficient over the GO Shape of the Thin Delta Wing.

Now the comparison is made between the GO shapes of FCs, with respect to minimum drag for two different values of the lift coefficient (which is taken as constraints for the optimization), at the same cruising Mach number and by the same other constraints.

If the chosen value of lift coefficient increases, the camber and the twist of the GO shape of FC increase in the central part of FC, and decrease in the vicinity of subsonic leading edges of FC. These influences are maximal at the rear part of the FC.

Further the comparison is made between the GO shapes of FCs, with respect to minimum drag for two different values of the pitching moment coefficient (which is taken as constraint for the optimization), at the same cruising Mach number and by the same other constraints.

If the pitching moment coefficient increases, the camber and twist increase in the vicinity of its leading edges of FC and decrease in its central part. These influences are opposite as those produced by the increasing of lift coefficient. It is to remark that lower and upper limits for the quotient lift/pitching moment coefficients exist. If the downwash of the thin component of FC is expressed in form of a superposition of homogeneous polynomes in two variables, these limits are the following:

$$\frac{2}{3} \leq \frac{C_m}{C_\ell} \leq \frac{N+1}{N+2}$$

Hereby N is the order of the homogeneous polynom of highest degree taken into consideration in the superposition.

4. Conclusions

The increase of cruising Mach number has a great influence over the optimal planform of the GO shape of FC. If the area of planform is constant, it produces an increase of maximal depth and a decrease of span. The multipoint design can be realized by morphing with the help of movable leading edge flaps.

The increase of lift increase the camber and twist in the central part of the FC's surface and decrease them along the leading edges and the increase of pitching moment produces opposite tendencies over the camber and the twist of the GO shape of FC as the lift.

The GO shapes of FC and their behaviors by variation of start values of optimization are similar to those of birds in gliding flight, because the nature optimizes too..

5. References

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