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Memories with Solomon Marcus

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Abstract: The present paper is dedicated to Solomon Marcus. Even from the high-school, I was interested in the work of Solomon Marcus in Mathematical Linguistics. Later, I had the opportunity to meet him personally, and to discuss with him about many topics. He was a polymath. At the Institute of Mathematics, we wrote a paper together. I also refereed an editorial paper about his work in 2021. His influence on my work extends to the present days. The current paper brings hot topics from our discussions to the interested reader: some topology conjectures, results about Boolean algebras and a self-dual theorem in geometry.

Keywords: Solomon Marcus, memories, mathematical linguistics.

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1 Introduction and a little history

Even from the last classes of high-school, I was fascinated by the work of Solomon Marcus (see [1]). I had participated in some communications sessions for high-school students, and I learned about the entropy of a translated poetry (see [2]). Formulas from the work of Solomon Marcus in Mathematical Linguistics heavily applied in Marius Iancu's communication at that time. My colleague and me, won the best prizes. Soon, I also became an author, by publishing an exercise in the Romanian Gazette. That exercise was something like this: *For a real number $a > 1$, we have the following limit:*

$$\lim_{n\to\infty} \frac{(a + \frac{1}{a^n})^n - a^n}{n} = \frac{1}{a} \quad . \tag{1.1}$$

Later, I extended my Romanian Gazette exercise, which thus grew up as an article (see [3]). Some other mathematicians impressed by that article wrote a book. That successful volume was then continued with a second volume, and some of these authors were invited at the best universities from Europe.

I had the opportunity to meet Solomon Marcus and discuss with him as a student and as a researcher at the Institute of Mathematics of the Romanian Academy. At the University of Bucharest, I followed him from some kind of distance. At the Institute of Mathematics, we met many times (see [2]). As a result of our "unexpected" meetings, we wrote a paper together (see [4]). We got some of these ideas after a long discussion on Euclidean geometry. It might be the case that Solomon Marcus was actually impressed by the paper [5], and he also wrote on transcendental numbers several papers during those years. I will refer to some remarks of [4] in the next section. Solomon Marcus also invited my friends and me to many events. I spent many hours in the libraries studying his works in Boolean algebras, informatics, literature and philosophy, trying to give a replica to his legacy (see [6, 7, 8]). Last week, I met one of the most important Romanian writers: Mircea Cartarescu. "What do you think about the connection between Mathematics and Literature ?", I asked him. He only said to me: "Solomon Marcus"...

In 2021, I was editor for an article about Solomon Marcus' work. His influence on my thinking and on my work extends to the present days. In the following sections, we will consider concrete subjects following from the above notes. Thus, the next section refers to our conjectures and the implications in topology, the third sections refers to Boolean algebras, and the last section deals with self-dual theorems in geometry. All these topics are in line with our meetings with Solomon Marcus, and, in some way, consequences of our discussions.

2 Remarks at recent conferences

A few days ago, I was attending a series of conferences on Intersection Cohomology, given by Laurentiu Maxim. I remembered some conjectures from the paper [4], and I improvised a beautiful problem, which was well-received. I will refer to enhanced version of our conjecture below. Solomon Marcus and me formulated it after a long discussion on Euclidean geometry in a late December, just before Christmas.

Remark 2.1. For a Jordan curve in the Euclidean plane, we consider the maximum diameter (D) and the smallest diameter passing through the corresponding center of mass (d).

(i) If L is the length of the given curve then: $\frac{L}{D} \leq \pi \leq \frac{L}{d}$.

(ii) Moreover, the first inequality becomes equality if and only if the second inequality becomes equality if and only if the given curve is a circle.

(iii) If the area of the domain inside the given curve is A, then $d\ D > A$.

(iv) The equation $x^2 - \frac{L}{2}x + A = 0$ is not completely solved; for example, if the given curve is an ellipse, solving this equation is an unsolved problem.

3 An Euler formula for Boolean Algebras

Solomon Marcus was very keen on the Euler identity. We had many meetings in which we have talked about on it. I am sure he would enjoy the following observations, which try to cast some properties of the Euler formula in the Boolean algebra setting.

Let $\mathcal{A} = (A, \vee, \wedge, 0, 1, \hat{})$ be a Boolean algebra. We consider a new Boolean algebra $\mathcal{B} = \mathcal{A} \times \mathcal{A} \times \mathcal{A}$, with the pointwise operations. For $X = (a, b, c) \in \mathcal{B}$, we define three functions, exp_B, $\cos_B$, $\sin_B : \mathcal{B} \to \mathcal{B}$, by the following formulas:

$$e^X = exp_B((a,\ b,\ c)) = (\hat{a}, \hat{b},\ 1) \in \mathcal{B},$$

$\cos(X) = \cos_B((a,b,c)) = (1, \hat{b}, \hat{c}) \in \mathcal{B}$ and $\sin(X) = \sin_B((a,b,c)) = (0,0,c) \in \mathcal{B}$.

The following properties hold for the above functions:

$$e^{(0,0,0)} = (\hat{0},\ \hat{0},\ 1) = (1,\ 1,\ 1) = \mathbf{1},$$

$$e^{X \vee Y} = (a_1 \hat{\vee} a_2,\ b_1 \hat{\vee} b_2,\ 1) = (\hat{a_1},\ \hat{b_1},\ 1) \wedge (\hat{a_2},\ \hat{b_2},\ 1) = e^X \wedge e^Y ; \tag{3.1}$$

$$\cos(0,0,0) = (1,\ \hat{0},\ \hat{0}) = \mathbf{1},$$

$$\cos(X \vee Y) = (1,\ b_1 \hat{\vee} b_2,\ c_1 \hat{\vee} c_2) = (1,\ \hat{b_1},\ \hat{c_1}) \wedge (1,\ \hat{b_2},\ \hat{c_2}) = \cos(X) \wedge \cos(Y) ; \tag{3.2}$$

$$\sin(0,0,0) = (0,0,0) = \mathbf{0},$$

$$\sin(X \vee Y) = [\sin(X) \wedge \cos(Y)] \vee [\cos(X) \wedge \sin(Y)] \vee [\sin(X) \wedge \sin(Y)] ; \tag{3.3}$$

$$\sin^{2\wedge}(X) \vee \cos^{2\wedge}(X) = [\sin(X) \wedge \sin(X)] \vee [\cos(X) \wedge \cos(X)] = (1,1,1) .$$

Theorem 3.1. *Let $J = (0,1,1) \in \mathcal{B}$. Then, the following Euler type equation holds:*

$$e^{J \wedge X} = \cos(X) \vee [\ J \wedge \sin(X)\ ]. \tag{3.4}$$

Proof. The proof of the formula (3.4) is direct. Let $e^{J\wedge X} = e^{(0,\ b,\ c)} = (1,\ \hat{b},\ 1)$. On the other hand $\cos X \vee [(0,1,1) \wedge \sin X] = (1, \hat{b}, \hat{c}) \vee (0,0,c) = (1, \hat{b}, 1)$.

□

Remark 3.2. Resembling the Euler identity, the following identity holds:

$$e^{J\wedge(1,0,0)} = \mathbf{1}. \tag{3.5}$$

Remark 3.3. One application of the formula (3.4) is a shorter proof for the fact that

$\cos(X\vee Y) \vee [\, J\wedge\sin(X\vee Y)\,] = [\,\cos(X) \vee [\, J\wedge\sin(X)]\,]\wedge[\,\cos(Y) \vee [\, J\wedge\sin(Y)]\,]$.

Indeed,

$\cos(X \vee Y) \ \vee\ [\ J \wedge \sin(X \vee Y)\ ] = e^{J\wedge(X\vee Y)} = e^{(J\wedge X)\vee(J\wedge Y)} = e^{J\wedge X} \wedge e^{J\wedge Y} = [\ \cos(X) \ \vee\ [\ J \wedge \sin(X)]\ ] \wedge [\ \cos(Y) \ \vee\ [\ J \wedge \sin(Y)]\ ]$.

Remark 3.4. As explained in the paper [4], the above remark could lead to solutions for the Yang-Baxter equations.

Remark 3.5. The applications of this approach are diverse. For example, by Hopf Algebras theory (the section on representative coalgebras), we are lead to a coalgebra structure: $\Delta(c) = c \otimes c,\ \ \varepsilon(c) = 1;\ \ \Delta(s) = s \otimes c + c \otimes s + s \otimes s,\ \ \varepsilon(s) = 0$.
The current Euler formula leads to the existence of a certain coideal.

4 A self-dual theorem in geometry

In general a theorem in Euclidean geometry is different from its dual. This happens because in the dual theorem we replace the points with lines and the lines become points. In this section we will find a counterexample for this general rule. As Solomon Marcus was passionate about the Euclidean geometry, the following theorem could have been included in our meeting discussions.

Let us fix the terminology first. Let $\mathcal{A}$ and $\mathcal{B}$ be two triangles with vertices A_1, A_2 and A_3, and, respectively with vertices B_1, B_2 and B_3. We will write $\mathcal{A} \ni\in \mathcal{B}$ if and only if $A_1 \in B_2B_3$ and $B_1 \in A_2A_3$.

Theorem 4.1. *Let* $\mathcal{O} \ni\in \mathcal{X}$, $\mathcal{O} \ni\in \mathcal{Y}$ *and* $\mathcal{O} \ni\in \mathcal{Z}$.
Let $\mathcal{X} \ni\in \mathcal{A}$, $\mathcal{X} \ni\in \mathcal{A}'$, $\mathcal{Y} \ni\in \mathcal{B}$, $\mathcal{Y} \ni\in \mathcal{B}'$, $\mathcal{Z} \ni\in \mathcal{C}$ *and* $\mathcal{Z} \ni\in \mathcal{C}'$.
Let $\mathcal{A} \ni\in \mathcal{M}$, $\mathcal{B} \ni\in \mathcal{M}$, $\mathcal{A}' \ni\in \mathcal{M}'$, $\mathcal{B}' \ni\in \mathcal{M}'$,
$\mathcal{A} \ni\in \mathcal{N}$, $\mathcal{C} \ni\in \mathcal{N}$, $\mathcal{A}' \ni\in \mathcal{N}'$, $\mathcal{C}' \ni\in \mathcal{N}'$,
$\mathcal{B} \ni\in \mathcal{S}$, $\mathcal{C} \ni\in \mathcal{S}$, $\mathcal{B}' \ni\in \mathcal{S}'$ *and* $\mathcal{C}' \ni\in \mathcal{S}'$.
Let $\mathcal{M} \ni\in \mathcal{P}$, $\mathcal{M}' \ni\in \mathcal{P}$, $\mathcal{N} \ni\in \mathcal{Q}$, $\mathcal{N}' \ni\in \mathcal{Q}$, $\mathcal{S} \ni\in \mathcal{R}$ *and* $\mathcal{S}' \ni\in \mathcal{R}$.
Then, there exists $\mathcal{O}'$ *such that* $\mathcal{P} \ni\in \mathcal{O}'$, $\mathcal{Q} \ni\in \mathcal{O}'$ *and* $\mathcal{R} \ni\in \mathcal{O}'$.

Proof. The proof follows from carefully applying the Desargues theorem and its dual. □

Remark 4.2. This is a self-dual theorem, because the dual of a triangle is also a triangle.

Remark 4.3. The above approach can be used to construct other self-dual theorems.

References

[1] *Meetings with Solomon Marcus*, Spandugino Publishing House (Eds. L. Spandonide and G. Paun), 2010.

[2] *Intalniri cu Solomon Marcus* , Spandugino Publishing House (Eds. L. Spandonide and G. Paun), 2011, volumul 2, 155-156.

[3] F. Nichita, Asupra unor limite remarcabile de functii, Gazeta Matematica, Seria B, No. 1 / 2007, 2-6.

[4] S. Marcus and F. Nichita, On transcendental numbers: new results and a little history, *Axioms*, 2018, **7**, 15.

[5] F. Nichita, On Transcendental Numbers, *Axioms*, 2014, **3**(1):64-69.

[6] F. Nichita, Poezie si Matematica. Nichita Stanescu si Solomon Marcus. Intalniri, *Caiete Critice* 2018, 3 (365), pag. 23 – 26.

[7] F. Nichita, Poezie si matematica (II), *Caiete Critice* 2018, **3**, 374, pag. 70 – 72.

[8] F. Nichita, (Ed.), Mathematics and Poetry, with a View towards Machine Learning, Sci, MDPI, Elvetia. https://www.mdpi.com/journal/sci/special_issues /Mathematics_Poetry_View_Machine_Learning

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