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SUPERNORMAL CONES AND ABSOLUTE SUMMABILITY

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1. Recently we defined in [7] an interesting class of convex cones imposed by the theory of Pareto optimum, by the study of critical points of dynamical systems and by the study of conical support points.

This convex cone was called in [7] "nuclear cone", since every normal cone in a nuclear space, is a nuclear cone in our sense.

But since recently, we observed that the nuclear cone, by its properties, seems to be a reinforcement of concept of normal cone, we call in this paper the nuclear cone, "Supernormal Cone".

Other applications of nuclear (or supernormal) cones, as for example, in the fixed point theory, we find in [8].

In this paper, which is a development of our paper [9], we give an interesting application of supernormal cones on summability of positive operators.

2. We use in this paper the concept of locally convex space defined by Treves [24], that is, a couple (E, Spec (E)), where E is a real vector space and Spec (E) is a set of seminorms on E such that,

1°) $(\forall\lambda \in R_+)(\forall p \in \text{Spec}(E))(\lambda p \in \text{Spec}(E))$,

2°) if $p \in \text{Spec}(E)$ and q is a seminorm on E such that $q \leq p$ then $q \in \text{Spec}(E)$,

3°) $(\forall p_1, p_2 \in \text{Spec}(E))(\sup(p_1,p_2) \in \text{Spec}(E))$ (where, $\sup(p_1,p_2)(x) = \sup(p_1(x),p_2(x))$ for any $x \in E$).

If Spec (E) is given, then there exists a locally convex topology τ on E, such that $E(\tau)$ is a locally convex vector space and a seminorm p on E is τ-continuous if and only if $p \in \text{Spec}(E)$.

A subset $\mathcal{B} \subset \text{Spec}(E)$ is called a basis of Spec (E), if and only if, for every $p \in \text{Spec}(E)$ there exists $q \in \mathcal{B}$ and a real number $\lambda > 0$ such that $p \leq \lambda q$.

We suppose that the Spec (E) has a Hausdorff basis, that is, $\text{Ker}\,\mathcal{B} = \{0\}$, where $\text{Ker}\,\mathcal{B} = \{x \in E \mid p(x) = 0,\ \forall p \in \mathcal{B}\}$.

For our terminology on nets we recommend [15] and for our terminology on convex cones we recommend [13], [17], [6].

If $(E, \text{Spec}(E))$ is a locally convex space, we denote by E' the topological dual of E.

A subset $K \subset E$ is called a convex cone if,

i) $K + K \subset K$

ii) $(\forall \lambda \in R_+)(\lambda K \subset K)$.

Let τ be the locally convex topology defined on E by the Spec (E).

We say that the convex cone $K \subset E$ is normal (with respect to τ) if, one of the following equivalent assertions are satisfied:

n_1). there exists a basis $\mathcal{B}$ of Spec (E) such that, $(\forall p \in \mathcal{B})$ $(\forall x,y \in K)(x \leq y \Rightarrow p(x) \leq p(y))$,

n_2). if $\{x_i\}_{i\in I}$, $\{y_i\}_{i\in I}$ are two nets of K such that, $(\forall i \in I)$ $(0 \leq x_i \leq y_i)$ and $\lim_{i\in I} y_i = 0$ then, $\lim_{i\in I} x_i = 0$.

For other properties of normal cones see [6], [13], [17].

Definition 1.

A convex cone $K \subset E$ is called τ-supernormal (or nuclear), if and only if, there exists a basis $\mathcal{B}$ of Spec (E) such that,

(1): $(\forall p \in \mathcal{B})(\exists f_p \in E')(\forall x \in K)(p(x) \leq f_p(x))$.

Proposition 1.

If $(E(\tau), \text{Spec }(E))$ is a Hausdorff locally convex space, then any τ-supernormal convex cone $K \subset E$ is a τ-normal cone.

Proof.

Indeed, let $\{x_i\}_{i\in I}$, $\{y_i\}_{i\in I}$ be two nets of K, such that, for any $i \in I$, $0 \leq x_i \leq y_i$, and $\lim_{i\in I} y_i = 0$.

Since K is a τ-supernormal cone, there exists a basis $\mathcal{B}$ of Spec (E) satisfying formula (1) and we have, $0 \leq p(x_i) \leq f_p(x_i) \leq f_p(y_i)$, for any $p \in \mathcal{B}$ and $i \in I$. [Formula (1) implies the positivity of f_p].

Now, since $\lim_{i\in I} f_p(y_i) = 0$ we obtain that $\lim_{i\in I} p(x_i) = 0$ and because $\mathcal{B}$ is a basis of Spec (E), we have $\lim_{i\in I} x_i = 0$, that is, K is a τ-normal cone.//

Proposition 2.

If $(E(\tau), \text{Spec }(E))$ is a Hausdorff locally convex space then, any τ-normal convex cone $K \subset E$ is $\sigma(E,E')$-supernormal.

Proof.

If p is a seminorm $\sigma(E,E')$-continue then there exists a constant $C > 0$ and $f_1, f_2, \dots, f_n \in E'$ such that, $p(x) \leq C \sup_{i=I}^{n} (|f_i|(x))$, for any $x \in E$.

Since K is a τ-normal cone, there exist for any $i = 1, 2, \dots, n$, $h_i, g_i \in K'$ such that $f_i = h_i - g_i$ and hence for any $x \in K$ we have,

$$p(x) \leq C \sup_{i=1}^{n} (|f_i|(x)) \leq C \sup_{i=1}^{n} ((|h_i| + |g_i|)(x)) =$$

$$= C \sup_{i=1}^{n} (h_i(x) + g_i(x)) \leq C \sum_{i=1}^{n} (h_i + g_i)(x),$$

that is, K is $\sigma(E,E')$-supernormal.//

Corollary.

A convex cone $K \subset E$ is $\sigma(E,E')$-supernormal, if and only if, it is $\sigma(E,E')$-normal.

Examples.

1°) A convex cone $K \subset E$ is called "well based" if, there exists a closed, convex, bounded set $A \subset E$ such that

b_1). $0 \notin \bar{A}$

b_2). $K = \bigcup_{\lambda \in R_+} \lambda A$

In any locally convex space $E(\tau)$, any well based convex cone is τ-supernormal [7].

2°) In a normed space $(E, \| \ \|)$ a convex cone $K \subset E$ is supernormal, if and only if, it is well based.

3°) In a locally convex space $(E(\tau), \text{Spec}(E))$ a locally compact (or weakly locally compact) convex cone $K \subset E$ is τ-supernormal.

4°) In a nuclear space $(E(\tau), \text{Spec}(E))$, a convex cone $K \subset E$ is τ-supernormal, if and only if, it is τ-normal. For nuclear spaces see [18], [25].

5°) Let $(E(\tau), \text{Spec}(E))$ be a locally convex space and let $\{f_n\}_{n \in N}$ be a sequence of continuous linear forms.

Consider $K \subset E$ a convex cone and suppose that f_n, for any $n \in N$, is positive with respect to the order defined by K.

The convex cone K is called <u>semicomplet</u> with respect to $\{f_n\}_{n\in N}$, if and only if, for any sequence $\{x_m\}_{m\in N} \subset K$ such that $\sum_{m=1}^{\infty} f_n(x_m) < +\infty$; $\forall\, n \in N$ we have that $\{x_m\}_{m\in N}$ is summable and $\sum_{m=1}^{\infty} x_m \in K$.

In [7] we proved that, if $K \subset E(\tau)$ is a semicomplet convex cone with respect to $\{f_n\}_{n\in N}$, then K is τ-supernormal.

6°) Let $(E(\tau), \mathrm{Spec}\,(E))$ be a Hausdorff locally convex space and suppose that $K \subset E$ is a convex cone such that $K \cap (-K) = \{0\}$. If K is $\sigma(E,E')$-complet, $\sigma(E,E')$-normal and there exists a countable fundamental system of weak neighborhoods of zero with respect to K, then K is a τ-supernormal cone.

7°) The convex cone $\mathcal{H}^+$ of positive harmonic functions on a locally compact space Ω, with respect to an axiomatic theory (Bauer, Brelot, Constantinescu-Cornea or Mokobodzki-Sibony) is a supernormal cone.

Other examples of supernormal cones we can obtain using the order topology [17], [13], [26].

Let $\langle E,F\rangle$ be a dual system of vector spaces and suppose that E is ordered by K and F by K° (the dual cone of K).

If E is generated by K then, the locally convex topology defined by the basis $\{[-f,f]^\circ\}_{f\in K^\circ}$ on E is called the <u>order topology</u>. We denote this topology by $\mathcal{O}(E,F)$.

If $F = E'$ and E is generated by K then the topology $\mathcal{O}(E,E')$ is defined by the family of seminorms $\{p_f\}_{f\in K^\circ}$, where, $p_f(x) = \sup\{|g(x)| \mid g\in[-f,f]\}$; $\forall x \in E$.

For other details on $\mathcal{O}(E,F)$ we recommend [26].

Proposition 3.

If $E(\tau)$ is a locally convex lattice [17] then $K = \{x \in E | x \geq 0\}$ is $\mathcal{O}(E,E')$-supernormal.

Proof.

Indeed, in this case $\{p_f\}_{f \in K^\circ}$ is a basis of Spec $(\mathcal{O}(E,E'))$ and it is well known [17, corollary 2.6] that $\mathcal{O}(E,E')$ is consistent with the duality $\langle E,E' \rangle$. Moreover, the definition of p_f implies, $(\forall x \in K)(p_f(x) \leq f(x))$.

Proposition 4.

Suppose that E is a regularly ordered vector space (that is, the points of E are separated by the order dual E^+).

If $K \subset E$ is a generating convex cone then, it is $\mathcal{O}(E,E^+)$-supernormal.

Proof.

In this case $\mathcal{O}(E,E^+)$ is well defined and consistent with the dual system $\langle E,E^+ \rangle$ [17].

Let $E(\tau)$ be an ordered locally convex space for which $K = \{x \in E | x \geq 0\}$ is a generating cone and let p be a seminorm on E.

We say that p is a (PL)-seminorm if there exists an $f \in K'$ such that,

(*): $p(x) \leq \inf \{f(w) | w \pm x \in K\}; \forall x \in E.$

Proposition 5.

Let $(E(\tau), \text{Spec } (E))$ be a locally solid space [17] and $K = \{x \in E | x \geq 0\}$.

If every equicontinuous subset of E' is ordered bounded, then K is τ-supernormal.

Proof.

From [26][Corollary 3.1.4, p. 109] we have in this case that, any continuous seminorm p on E is a (PL)-seminorm and hence f satisfies the formula (*).

A locally convex space $E(\tau)$ is an (L)-space if, it is a locally convex lattice possessing a Spec (E) with a basis $\mathcal{B}$ such that for any $p \in \mathcal{B}$ the following properties are satisfied:

ℓ_1) $(\forall\, x, y \in E)(|x| \leq |y| \Rightarrow p(x) \leq p(y))$

ℓ_2) $(\forall\, x, y \in K)(p(x+y) = p(x) + p(y))$

The (L)-spaces was studied in [2], [5], [20], [4].

Proposition 6.

If $(E(\tau)$, Spec $(E))$ is an (L)-space, then $K = \{x \in E \mid x \geq 0\}$ is a τ-supernormal cone.

Proof.

In this case, $E(\tau)$ is a locally solid space and if $p \in \mathcal{B}$ where $\mathcal{B}$ is a basis of the Spec (E) satisfying ℓ_1) and ℓ_2), then from [26][Corollary 3.26, p. 131] we have that p is a (PL)-seminorm, which implies (as in the proof of Prop. 5) that K is τ-supernormal.

3. Using the supernormal cones, we study now in this paragraph, some properties of summable families in a locally convex spaces.

Let $\{x_i\}_{i \in I}$ be a family of elements of locally convex space $(E(\tau)$, Spec $(E))$ and we denote by $\mathcal{F}(I)$ the family of all finite subset of I. Consider on $\mathcal{F}(I)$ the order relation defined by inclusion.

We say that a family $\{x_i\}_{i \in I} \subset E$ is τ-summable if the associate net

$\{\delta_{\underline{i}}\}_{\underline{i}\in F(I)}$, where $\delta_{\underline{i}} = \sum_{i\in\underline{i}} x_i$, is τ-convergent.

This concept is the well known concept of unconditional summability, which was studied in [3], [14], [21-23] and used in functional analysis and measure theory [1], [18], [21-22], [3], [14].

If a family $\{x_i\}_{i\in F(I)}$ is τ-summable we denote, $\sum_{i\in I} x_i \dot{<} +\infty$.

A family $\{x_i\}_{i\in I} \subset E$ is τ-summable and $\sum_{i\in I} x_i = s \in E$, if and only if, for any 0-neighborhood U in E there exists $\underline{i}_o \in F(I)$ such that $\delta_{\underline{i}} \in s + U$, for any $\underline{i} \supset \underline{i}_o$, $\underline{i} \in F(I)$.

Let $K \subset E$ be a convex cone and $\{x_i\}_{i\in I}$ a family of elements of E.

<u>Definition 2.</u>

A family $\{x_i\}_{i\in I} \subset E$ is called <u>(OA)-Cauchy</u> if:

1°) $(\forall\, i\in I)(\exists\, u_i, v_i\in K)(x_i = u_i - v_i)$

2°) the nets $\{\delta^u_{\underline{i}}\}_{\underline{i}\in F(I)}$, $\{\delta^v_{\underline{i}}\}_{\underline{i}\in F(I)}$, where $\delta^u_{\underline{i}} = \sum_{i\in\underline{i}} u_i$; $\delta^v_{\underline{i}} = \sum_{i\in\underline{i}} v_i$; are Cauchy nets, that is, for any 0-neighborhood U in E there exist $\underline{i}^u_o$, $\underline{i}^v_o \in F(I)$ such that, $\delta^u_{\underline{i}}$, $\delta^v_{\underline{i}} \in U$ for all $\underline{i},\underline{j} \in F(I)$ such that $\underline{i} \cap \underline{i}^u_o = \phi$ and $\underline{j} \cap \underline{i}^v_o = \phi$.

If $\{x_i\}_{i\in I}$ is a (OA)-Cauchy family in E, then $\{x_i\}_{i\in I}$, $\{v_i\}_{i\in I}$ are unconditional Cauchy in the sense of concept used in [21].

A family $\{x_i\}_{i\in I} \subset E$ is unconditional Cauchy, if and only if $\{\delta_{\underline{i}}\}_{\underline{i} \in F(I)}$ is precompact in E [21-22].

Recall that, a family of real numbers $\{\xi_i\}_{i\in I}$ is summable, if and only if, there exists $\rho \in R$, $\rho > 0$ such that

$(\forall\, \underline{i} \in F(I))(\sum_{i\in\underline{i}} |\xi_i| \leq \rho)$, [18].

Definition 3.

A family $\{x_i\}_{i\in I} \subset E$ is called absolutely summable if, for any 0-neighborhood U in E we have, $\sum_{i\in I} p_U(x_i) < \infty$, where p_U is the Minkovski functional of U.

Theorem 1.

Let $(E(\tau), \text{Spec}(E))$ be a Hausdorff locally convex space. If $K \subset E$ is a τ-supernormal convex cone, then any (OA)-Cauchy family $\{x_i\}_{i\in I} \subset E$ is τ-absolutely summable.

Proof.

Since $\{x_i\}_{i\in I}$, is a (OA)-Cauchy family, we have that

$\{\delta^u_{\underline{i}}\}_{\underline{i}\in F(I)}$, $\{\delta^v_{\underline{i}}\}_{\underline{i}\in F(I)}$ are Cauchy nets.

Using the τ-supernormality of K we can choose a basis $\mathcal{B}$ of Spec (E) such that,

$(\forall\ p \in \mathcal{B})(\exists\ f_p \in K')(\forall\ x \in K)(p(x) \leq f_p(x))$.

Since, a continuous linear application, transform a basis of Cauchy filter in a basis of Cauchy filter and since R is complete we have that, $\{f_p(u_i)\}_{i\in I}$, $\{f_p(v_i)\}_{i\in I}$ (where, for any $i \in I$, $x_i = u_i - v_i$) are summable families.

Hence, there exist $a_p, b_p \in R_+\setminus\{0\}$ such that,

$(\forall\ \underline{i} \in F(I))(\sum_{i\in\underline{i}} f_p(u_i) \leq a_p;\ \sum_{i\in\underline{i}} f_p(v_i) \leq b_p)$

and using the τ-supernormality of K we have,

$(\forall \underline{i} \in F(I))(\sum_{i\in\underline{i}} p(u_i) \leq \sum_{i\in\underline{i}} f_p(u_i) \leq a_p)$ and

$(\forall \underline{i} \in F(I))(\sum_{i\in\underline{i}} p(v_i) \leq \sum_{i\in\underline{i}} f_p(v_i) \leq b_p)$.

The last relations imply that, for any $p \in \mathcal{B}$ the families $\{p(u_i)\}_{i\in I}$,

$\{p(v_i)\}_{i\in I}$ are τ-summable and hence, for any $p \in \mathcal{B}$ $\{p(x_i)\}_{i\in I}$ are τ-summable.

Now, because for any 0-neighborhood U in E the Minkovski seminorm p_U is dominated by a seminorm of the form λp, where $\lambda > 0$, $\lambda \in R$ and $p \in \mathcal{B}$, we obtain that, $\{x_i\}_{i\in I}$ is τ-absolutely summable.

Corollary 1.

Suppose the assumptions of Theorem 1 and consider a countable family $\{x_n\}_{n\in N} \subset E$ such that, for any $n \in N$, $x_n = u_n - v_n$, $u_n, v_n \in K$.

If $\{u_n\}_{n\in N}$, $\{v_n\}_{n\in N}$ are $\sigma(E,E')$-summable then $\{x_n\}_{n\in N}$ is τ-summable and τ-absolutely summable.

Corollary 2.

Let $(E(\tau)$, Spec $(E))$ be a Hausdorff locally convex space and let $K \subset E$ be a τ-supernormal cone.

If a family $\{x_i\}_{i\in I} \subset E$ satisfies the property that, $\forall\, i \in I$ there exist $u_i, v_i \in K$ such that, $x_i = u_i - v_i$ and $\{u_i\}_{i\in I}$, $\{v_i\}_{i\in I}$ are $\sigma(E,E')$-summable, then $\{x_i\}_{i\in I}$ is τ-absolutely summable.//

Proof.

Indeed, since K is τ-supernormal it is τ-normal and from [17]-[Prop. 3.4, p. 91] we have that $\{x_i\}_{i\in I}$ is a (OA)-Cauchy family. Hence, Theorem 1 implies that $\{x_i\}_{i\in I}$ is τ-absolutely summable.

Remark.

A supernormal cone K, in a locally convex space E, has the Dvoretzky-Rogers property, that is, << any summable family of K is absolutely summable.

Consider now, two locally convex spaces (E, Spec (E)), (F, Spec (F)) and $T: E \to F$ a continuous linear mapping.

Definition 4.

We say that T is absolutely summable, if for any summable family $\{x_i\}_{i\in I} \subset E$ the family $\{T(x_i)\}_{i\in I}$ is absolutely summable.

Some properties of absolutely summable mappings we find in [18], [25].

Theorem 2.

Suppose E a vector lattice endowed with two locally convex topologies τ_1, τ_2 such that, for any τ_1-0-neighborhood U there exists a τ_2-0-neighborhood V having the properties:

1) V is convex, solid and sublattice of E

2) $V \subset U$

If $F(\tau_o)$ is a locally convex space, ordered by a τ_o-supernormal cone K, then any τ_1-continuous, positive, linear mapping $T: E \to F$ is τ_2-absolutely summable.

Proof.

Suppose $\{x_i\}_{i\in I} \subset E$ a τ_2-summable family and observe that $\{\delta_{\underline{i}}\}_{\underline{i}\in F(I)}$ (where $\delta_{\underline{i}} = \sum_{i\in\underline{i}} x_i$) is a τ_2-Cauchy net.

If U is a τ_1-0-neighborhood in E, then there exists a convex, solid and sublattice τ_2-0-neighborhood V such that $V \subset U$.

Since $\{x_i\}_{i\in I}$ is τ_2-summable, there exists $\underline{i}_o \in F(I)$ such that, for any $\underline{i} \in F(I)$ satisfying, $\underline{i} \cap \underline{i}_o = \phi$ we have, $\delta_{\underline{i}} \in V$.

For $\underline{i} \in F(I)$ such that $\underline{i} \cap \underline{i}_o = \phi$ we denote, $\underline{i} = \{i_1, i_2, \ldots, i_n\}$.

Certainly, for every $\underline{j} \subset \underline{i}$ we have also, $\delta_{\underline{j}} \in V$,

Denote by F_n the family of subsets of $\{i_1, i_2, \ldots, i_n\}$ and for $\underline{j} \in F_n$ put, $\delta_{\underline{j}} = \sum_{i \in \underline{j}} x_i$ (with $\delta_\phi = 0$).

We prove now the following formula,

$$(\alpha): \quad x_{i_1}^+ + x_{i_2}^+ + \ldots + x_{i_n}^+ = \sup \{\delta_{\underline{j}} | \underline{j} \in F_n\}.$$

Indeed, for $n = 1$ formula (α) is true since, $x_{i_1}^+ = \sup (0, x_{i_1})$.

Supposing now, formula (α) true for $n - 1$ we denote,

$$u_k = x_{i_1}^+ + \ldots + x_{i_k}^+ ; \; v_k = \sup \{\delta_{\underline{j}} | \underline{j} \in F_k\}.$$

From induction hypothesis we have, $u_{n-1} = v_{n-1}$.

Hence, we observe that,

i). $u_{n-1} \leq v_n$

ii). $u_{n-1} + x_{i_n} = v_{n-1} + x_{i_n} \leq v_n$

which imply, $0 \leq v_n - u_{n-1}$; $x_{i_n} \leq v_n - u_{n-1}$ and consequently, $x_{i_n}^+ \leq v_n - u_{n-1}$.

So, we obtain, $u_n \leq v_n$ and since $v_n \leq u_n$ we have, $u_n = v_n$, that is, formula (α) is true.

Since V is a solid sublattice we have, $\sum_{i \in \underline{i}} x_i^+, \sum_{i \in \underline{i}} x_i^- \in V$, for any $\underline{i} \in F(I)$ satisfying $\underline{i} \cap \underline{i}_o = \phi$, that is, $\{x_i^+\}_{i \in I}$, $\{x_i^-\}_{i \in I}$ are τ_1-unconditional Cauchy families.

The last result implies that $\{x_i\}_{i \in I}$ is a τ_1-(OA)-Cauchy family and hence, $\{T(x_i)\}_{i \in I}$ is a (OA)-Cauchy family in F, which implies, using Theorem 1, that $\{T(x_i)\}_{i \in I}$ is absolutely summable.//

A <u>locally convex lattice</u> is a vector lattice E endowed with a Hausdorff locally convex topology such that its topological dual space is a solid vector subspace of E^+.

<u>Definition 5</u>. [1]

A quasi-(M)-space is a locally convex lattice E such that, for any weak 0-neighborhood U in E there exists a solid directed 0-neighborhood for the Mackey topology of E which is enclosed by U.

<u>Corollary 1</u>.

Let E be a quasi-(M)-space and let F be a locally convex space ordered by a supernormal cone.

If $T: E \to F$ is a positive, linear mapping continuous from the weak topology of E to the strong topology of F then T is Mackey-absolutely summable.

<u>Definition 6</u>. [1]

An (M)-space is a locally convex lattice possessing a fundamental system of solid directed 0-neighborhoods.

<u>Remark</u>.

A locally convex lattice E is an (M)-space, if and only if, its topology may be generated by a spectre possessing a basis B such that, $p(\sup(|x|,|y|)) = \sup (p(x),p(y))$; $\forall p \in \mathcal{B}$ and any $x,y \in E$.

The (M)-spaces was studied in [1], [2], [5], [12], [19].

<u>Corollary 2</u>.

Let E be an (M)-space and let F be a locally convex space ordered by a supernormal cone.

If $T: E \to F$ is a continuous, positive linear mapping then it is absolutely summable.

Corollary 3.

If E is an (M)-space and F an (L)-space then, any continuous, positive linear mapping $T: E \to F$ is absolutely summable.

Corollary 4.

If E is an (M)-space and F a nuclear space ordered by a normal cone then any continuous, positive linear mapping $T: E \to F$ is absolutely summable.

Corollary 5.

Let E be an (M)-space and let F be an ordered locally convex space, locally solid such that any equicontinuous subset of F' is ordered bounded.

If $T: E \to F$ is a continuous, positive linear mapping then it is absolutely summable .

Corollary 6.

If E is an (M)-space and F a locally convex lattice then any continuous, positive linear mapping $T: E \to F$ is absolutely summable with respect to $\mathcal{O}(F,F')$.

In [19] Popa proved the following result, << if E is a separable locally convex lattice, topological complete, possessing both, an (L)-structure and an (M)-structure, then E is a nuclear space >> .

We consider interesting to know, if this result remains true substituting the (L)-structure by an order defined by a supernormal cone.

REFERENCES

[1] Constantinescu, C. *Spaces of Measures* (New York: Walter de Gruyter, 1984).

[2] ________________. "Weakly compact sets in locally convex vector lattices", Rev. Roum. Math. Pures et Appl. Tom. 14 (1969), 325-351.

[3] Dvoretzky A. and Rogers, C.A. "Absolute and unconditional convergence in normed linear spaces", Proc. Nat. Acad. Sci. 36 (1950), 192-197.

[4] Grigore, Gh. "La représentation des espaces réticulés localement convexes de type (L)", St. cerc. Math. Tom 22 (1970), 1182-1188.

[5] Isac, G. "The (M)-(L)-type duality for locally convex lattices", Rev. Roum. Math. Pures et Appl. Tom 16 (1971), 217-223.

[6] ______. "Cônes localement bornés et cônes complètement réguliers", Applications à l'analyse non-linéaire. Séminaire d'Analyse moderne. Univ. de Sherbrooke, (1980), 1-168.

[7] ______. "Sur l'existence de l'optimum de Pareto", Revista di Mat. Univ. di Parma 4 (1983), 303 - 325 .

[8] ______. "Un théorèm de point fixe de type Caristi dans les espaces localement convexes", Applications [To appear, Colloquium Math.].

[9] ______. "Un critère de sommabilité absolue dans les espaces localement convexes ordonnés." Cônes nucléaires. Mathematica Tom 15 (1983), 159-169.

[10] Isac, G. et Ichim I. "Quelques critères de normalité pour des cônes dans les espaces localement convexes", Rev. Roum. Math. Pures et Appl. Tom. 17 (1972), 1633-1637.

[11] Jameson, G.J.O. "Unconditional convergence in partially ordered linear spaces", Math. Ann. 200 (1973), 227-233.

[12] ____________. "Topological (M)-space", Math. Zeit. 103 (1968).

[13] ____________. *Ordered Linear Spaces* (Springer-Verlag, 1970).

[14] Macphail, M.S. "Absolute and unconditional convergence", Bull. Amer. Math. Soc. 53 (1947), 121-123.

[15] Marinescu, Gh. *Espaces Vectoriels Pseudotopologiques et Théorie des Distributions* (Wissenschaften: VEB Deutscher Verlag, 1963).

[16] Mokobodzki, G. "Cônes normaux et espaces nucléaires. Cônes semi-couplets. Séminaire Choquet - Initiation à l'analyse. 7e annee 1967/1968.

[17] Peressini, A.L. *Ordered Topological Vector Spaces* (New York: Harper & Row, 1967).

[18] Pietch, A. *Nuclear Locally Convex Spaces* (Springer Verlag, 1972).

[19] Popa, N. "Un critère de nucléarité pour des treillis", C.R. Acad. Sc. Paris t. 269 (1969), A. 355 - A. 356.

[20] ______. "Le produit tensoriel de deux (AL)-espaces." C.R. Acad. Sc. Paris t. 268 (1969), A. 1261 - A. 1263.

[21] Robertson, A.P. "Unconditional convergence and Vitali-Hahn-Saks theorem", Bull. Soc. Math. France. Memoire 31-32 (1972), 335-341.

[22] ____________. "On conditional convergence in topological vector spaces", Proc. Roy. Soc. Edinburgh Sect. A.68 (1969), 145-157.

[23] Rolewicz, S. and Ryll-Nardzewski, C. "On unconditional convergence in linear metric spaces", Colloq. Math. 17 (1967), 327-331.

[24] Treves, Fr. *Locally Convex Spaces and Linear Partial Differential Equations* (New York: Springer-Verlag, 1967).

[25] Wong, S. CH. *Schwartz Spaces, Nuclear Spaces and Tensor Products* (New York: Springer-Verlag, 1979).

[26] _________. *Convergence on Order-Bounded Sets* (New York: Springer-Verlag, 1976).