LIBERTAS MATHEMATICA vol XXVI (2006)

On θ-m-Continuous Functions

Takashi NOIRI and Valeriu POPA

Abstract. We introduce the notion of θ-m-continuous functions as functions from a set satisfying some minimal conditions into a topological space. We obtain several characterizations and properties of such functions. The functions enable us to formulate a unified theory of θ-continuity [16], θ-semi-continuity [6], θ-precontinuity [28].

2000 Mathematics Subject Classification: 54C08.

Keywords: m-structure, m-open, weakly m-continuous, θ-m-continuous, m-continuous, θ-continuous, θ-semi-continuous, θ-precontinuous.

1 Introduction

Semi-open sets, preopen sets, α-open sets, β-open sets and δ-open sets play an important role in the researches of generalizations of continuity in topological spaces. By using these sets many authors introduced and studied various types of modifications of continuity. Fomin [16] introduced the notion of θ-continuous functions. Many properties of θ-continuous functions are studied in [15], [18], [19], [25], [26], [37] and other papers. In 1966, Andrew and Whittlesy [3] introduced the notion of closure continuity which is equivalent to θ-continuity. Some properties of closure continuity are studied in [8] and [9]. Arya and Bhamini [6] introduced the concept of θ-semi-continuous functions which are studied in [27] and [17]. Recently, the notion of θ-precontinuous functions is introduced and investigated in [28]. Quite recently, Baker [7] has introduced the notion of weakly θ-precontinuous functions. However, this notion is quite same with one of θ-precontinuous functions. On the other hand, the present authors [38] introduced and studied the notion of m-continuous functions.

In this paper, in order to unify several characterizations of θ-continuous functions, θ-semi-continuous functions and θ-precontinuous functions, we introduce the notion of θ-m-continuous functions which are functions from a set satisfying some minimal conditions into a topological spaces.

2 Preliminaries

Let (X, τ) be a topological space and A a subset of X. The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A is said to be *regular closed* (resp. *regular open*) if $\mathrm{Cl}(\mathrm{Int}(A)) = A$ (resp. $\mathrm{Int}(\mathrm{Cl}(A)) = A$). A subset A is said to be *δ-open* [47] if for each $x \in A$ there exists a regular open set G such that $x \in G \subset A$. A point $x \in X$ is called a *δ-cluster* point of A if $\mathrm{Int}(\mathrm{Cl}(V)) \cap A \neq \emptyset$ for every open set V containing x. The set of all δ-cluster points of A is called the *δ-closure* of A and is denoted by $\mathrm{Cl}_\delta(A)$. The set $\{x \in X : x \in U \subset A$ for some regular open set U of $X\}$ is called the *δ-interior* of A and is denoted by $\mathrm{Int}_\delta(A)$.

The θ-closure of A, denoted by $\mathrm{Cl}_\theta(A)$, is defined as the set of all $x \in X$ such that $\mathrm{Cl}(V) \cap A \neq \emptyset$ for every open set V containing x. If $A = \mathrm{Cl}_\theta(A)$, then A is said to be *θ-closed* [47]. The complement of a θ-closed set is said to be *θ-open*. It is shown in [47] that $\mathrm{Cl}_\theta(V) = \mathrm{Cl}(V)$ for every open set V of X and $\mathrm{Cl}_\theta(S)$ is closed in X for every subset S of X.

Definition 2.1. Let (X, τ) be a topological space. A subset A of X is said to be

(1) *semi-open* [20] (resp. *preopen* [22], *α-open* [24], *β-open* [1] or *semi-preopen* [4]) if $A \subset \mathrm{Cl}(\mathrm{Int}(A))$, (resp. $A \subset \mathrm{Int}(\mathrm{Cl}(A))$, $A \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$, $A \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$),

(2) *δ-preopen* [43] (resp. *δ-semi-open* [36]) if $A \subset \mathrm{Int}(\mathrm{Cl}_\delta(A))$ (resp. $A \subset \mathrm{Cl}(\mathrm{Int}_\delta(A))$).

The family of all semi-open (resp. preopen, α-open, β-open, δ-preopen, δ-semi-open) sets in X is denoted by $\mathrm{SO}(X)$ (resp. $\mathrm{PO}(X)$, $\alpha(X)$, $\beta(X)$, $\delta\mathrm{PO}(X)$, $\delta\mathrm{SO}(X)$).

Definition 2.2. The complement of a semi-open (resp. preopen, α-open, β-open, δ-preopen, δ-semi-open) set is said to be *semi-closed* [10] (resp. *preclosed* [22], *α-closed* [23], *β-closed* [1] or *semi-preclosed* [4], *δ-preclosed* [43], *δ-semi-closed* [36]).

Definition 2.3. The intersection of all semi-closed (resp. preclosed, α-closed, β-closed, δ-preclosed, δ-semi-closed) sets of X containing A is called the *semi-closure* [10] (resp. *preclosure* [13], *α-closure* [23], *β-closure* [2] or *semi-preclosure* [4], *δ-preclosure* [43], *δ-semi-closure* [36]) of A and is denoted by $\mathrm{sCl}(A)$ (resp. $\mathrm{pCl}(A)$, $\alpha\,\mathrm{Cl}(A)$, ${}_\beta\,\mathrm{Cl}(A)$ or $\mathrm{spCl}(A)$, $\mathrm{pCl}_\delta(A)$, $\mathrm{sCl}_\delta(A)$).

Definition 2.4. The union of all semi-open (resp. preopen, α-open, β-open, δ-preopen, δ-semi-open) sets of X contained in A is called the *semi-interior* (resp. *preinterior*, *α-interior*, *β-interior* or *semi-preinterior*, *δ-preinterior*, *δ-semi-interior*) of A and is denoted by $\mathrm{sInt}(A)$ (resp. $\mathrm{pInt}(A)$, $\alpha\,\mathrm{Int}(A)$, ${}_\beta\,\mathrm{Int}(A)$ or $\mathrm{spInt}(A)$, $\mathrm{pInt}_\delta(A)$, $\mathrm{sInt}_\delta(A)$).

Throughout the present paper, (X, τ) and (Y, σ) denote topological spaces and $f : (X, \tau) \to (Y, \sigma)$ presents a (single valued) function.

Definition 2.5. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be *θ-continuous* [16] or *closure-continuous* [3] (resp. *θ-semi-continuous* [6], *θ-precontinuous* [28]) at $x \in X$ if for each open set V of Y containing $f(x)$, there exists an open (resp. semi-open, preopen) set U of X containing x such that $f(\mathrm{Cl}(U)) \subset \mathrm{Cl}(V)$ (resp. $f(\mathrm{sCl}(U)) \subset \mathrm{Cl}(V)$, $f(\mathrm{pCl}(U)) \subset \mathrm{Cl}(V)$). A function $f : (X, \tau) \to (Y, \sigma)$ is said to be *θ-continuous* (resp. *θ-semi-continuous*, *θ-precontinuous*) if it has this property at each $x \in X$.

3 θ-m-Continuous Functions

Definition 3.1. A subfamily m_X of the power set $\mathcal{P}(X)$ of a nonempty set X is called a *minimal structure* (briefly *m-structure*) on X if $\emptyset \in m_X$ and $X \in m_X$. By (X, m_X), we denote a nonempty set X with a minimal structure m_X on X and call it an m-space. Each member of m_X is said to be *m_X-open* (or briefly *m-open*) and the complement of an m_X-open set is said to be *m_X-closed* (or briefly *m-closed*).

Remark 3.1. Let (X, τ) be a topological space. Then the families τ, $\mathrm{SO}(X)$, $\mathrm{PO}(X)$, $\alpha(X)$, $\beta(X)$, $\delta\mathrm{PO}(X)$ and $\delta\mathrm{SO}(X)$ are all m-structures on X.

Definition 3.2. Let (X, m_X) be an m-space. For a subset A of X, the *m-closure* of A and the *m-interior* of A are defined in [21] as follows:

(1) $\mathrm{mCl}(A) = \bigcap\{F : A \subset F, X - F \in m_X\}$,

(2) $\mathrm{mInt}(A) = \bigcup\{U : U \subset A, U \in m_X\}$.

Remark 3.2. Let (X, τ) be a topological space and A a subset of X. If $m_X = \tau$ (resp. $\mathrm{SO}(X)$, $\mathrm{PO}(X)$, $\alpha(X)$, $\beta(X)$, $\delta\mathrm{PO}(X)$, $\delta\mathrm{SO}(X)$), then we have

(1) $\mathrm{mCl}(A) = \mathrm{Cl}(A)$ (resp. $\mathrm{sCl}(A)$, $\mathrm{pCl}(A)$, $\alpha\,\mathrm{Cl}(A)$, ${}_\beta\,\mathrm{Cl}(A)$, $\mathrm{pCl}_\delta(A)$, $\mathrm{sCl}_\delta(A)$),

(2) $\mathrm{mInt}(A) = \mathrm{Int}(A)$ (resp. $\mathrm{sInt}(A)$, $\mathrm{pInt}(A)$, $\alpha\,\mathrm{Int}(A)$, ${}_\beta\,\mathrm{Int}(A)$, $\mathrm{pInt}_\delta(A)$, $\mathrm{sInt}_\delta(A)$).

Lemma 3.1. (Maki et al. [21]) *Let (X, m_X) be an m-space. For subsets A and B of X, the following properties hold:*

(1) $\mathrm{mCl}(X - A) = X - \mathrm{mInt}(A)$ *and* $\mathrm{mInt}(X - A) = X - \mathrm{mCl}(A)$,

(2) *If* $(X - A) \in m$, *then* $\mathrm{mCl}(A) = A$ *and if* $A \in m$, *then* $\mathrm{mInt}(A) = A$,

(3) $\mathrm{mCl}(\emptyset) = \emptyset$, $\mathrm{mCl}(X) = X$, $\mathrm{mInt}(\emptyset) = \emptyset$ *and* $\mathrm{mInt}(X) = X$,

(4) *If* $A \subset B$, *then* $\mathrm{mCl}(A) \subset \mathrm{mCl}(B)$ *and* $\mathrm{mInt}(A) \subset \mathrm{mInt}(B)$,

(5) $A \subset \mathrm{mCl}(A)$ *and* $\mathrm{mInt}(A) \subset A$,

(6) $\mathrm{mCl}(\mathrm{mCl}(A)) = \mathrm{mCl}(A)$ *and* $\mathrm{mInt}(\mathrm{mInt}(A)) = \mathrm{mInt}(A)$.

Definition 3.3. A function $f : (X, m_X) \to (Y, \sigma)$, where (X, m_X) is an m-space and (Y, σ) is a topological space, is said to be *m-continuous* [38] (resp. *almost m-continuous* [41], *weakly m-continuous* [40]) at $x \in X$ if for each open set V of Y containing $f(x)$, there exists $U \in m_X$ containing x such that $f(U) \subset V$ (resp. $f(U) \subset \mathrm{Int}(\mathrm{Cl}(V))$, $f(U) \subset \mathrm{Cl}(V)$). A function $f : (X, m_X) \to (Y, \sigma)$ is said to be *m-continuous* (resp. *almost m-continuous*, *weakly m-continuous*) if it has the property at each point $x \in X$.

Lemma 3.2. (Popa and Noiri [38]) *For a function $f : (X, m_X) \to (Y, \sigma)$, the following properties are equivalent:*

(1) *f is m-continuous;*

(2) $f^{-1}(V) = \mathrm{mInt}(f^{-1}(V))$ *for every open set V of Y;*

(3) $f(\mathrm{mCl}(A)) \subset \mathrm{Cl}(f(A))$ *for every subset A of X;*

(4) $\mathrm{mCl}(f^{-1}(B)) \subset f^{-1}(\mathrm{Cl}(B))$ *for every subset B of Y;*

(5) $f^{-1}(\mathrm{Int}(B)) \subset \mathrm{mInt}(f^{-1}(B))$ *for every subset B of Y;*

(6) $\mathrm{mCl}(f^{-1}(K)) = f^{-1}(K)$ *for every closed set K of Y.*

Definition 3.4. A function $f : (X, m_X) \to (Y, \sigma)$, where (X, m_X) is an m-space and (Y, σ) is a topological space, is said to be *θ-m-continuous* at $x \in X$ if for each open set V of Y containing $f(x)$, there exists $U \in m_X$ containing x such that $f(\mathrm{mCl}(U)) \subset \mathrm{Cl}(V)$. A function $f : (X, m_X) \to (Y, \sigma)$ is said to be *θ-m-continuous* if it has the property at each point $x \in X$.

Remark 3.3. Let $f : (X, \tau) \to (Y, \sigma)$ be a function. If $m_X = \tau$ (resp. SO(X), PO(X)) and $f : (X, m_X) \to (Y, \sigma)$ is θ-m-continuous, then f is θ-continuous (resp. θ-semi-continuous, θ-precontinuous).

Definition 3.5. Let S be a subset of an m-space (X, m_X). A point $x \in X$ is called

(1) an m_θ*-adherent* point of S if $\mathrm{mCl}(U) \cap S \neq \emptyset$ for every $U \in m_X$ containing x,

(2) an m_θ*-interior* point of S if $\mathrm{mCl}(U) \subset S$ for some $U \in m_X$ containing x.

The set of all m_θ-adherent points of S is called the m_θ*-closure* of S and is denoted by $\mathrm{mCl}_\theta(S)$. If $A = \mathrm{mCl}_\theta(A)$, then A is called m_θ*-closed*. The complement of an m_θ-closed set is said to be m_θ*-open*. The set of all m_θ-interior points of S is called the m_θ*-interior* of S and is denoted by $\mathrm{mInt}_\theta(S)$.

Remark 3.4. Let (X, τ) be a topological space and $m_X = \tau$ (resp. $SO(X), PO(X)$), $\beta(X)$), then $\mathrm{mCl}_\theta(S) = \mathrm{Cl}_\theta(S)$ [47] (resp. $\mathrm{sCl}_\theta(S)$ [11], $\mathrm{pCl}_\theta(S)$ [33], $\mathrm{spCl}_\theta(S)$ [29]).

Lemma 3.3. (Noiri and Popa [30]) *Let (X, m_X) be an m-space and A, B subsets of X. Then the following properties hold:*

(1) $X - \mathrm{mCl}_\theta(A) = \mathrm{mInt}_\theta(X - A)$ *and* $X - \mathrm{mInt}_\theta(A) = \mathrm{mCl}_\theta(X - A)$,

(2) *A is m-θ-open if and only if* $A = \mathrm{mInt}_\theta(A)$,

(3) $A \subset \mathrm{mCl}(A) \subset \mathrm{mCl}_\theta(A)$ *and* $\mathrm{mInt}_\theta(A) \subset \mathrm{mInt}(A) \subset A$,

(4) *If $A \subset B$, then* $\mathrm{mCl}_\theta(A) \subset \mathrm{mCl}_\theta(B)$ *and* $\mathrm{mInt}_\theta(A) \subset \mathrm{mInt}_\theta(B)$,

(5) *If A is m_X-open, then* $\mathrm{mCl}(A) = \mathrm{mCl}_\theta(A)$.

Theorem 3.1. *For a function $f : (X, m_X) \to (Y, \sigma)$, the following properties are equivalent:*

(1) *f is θ-m-continuous;*

(2) $\mathrm{mCl}_\theta(f^{-1}(B)) \subset f^{-1}(\mathrm{Cl}_\theta(B))$ *for every subset B of Y;*

(3) $\mathrm{mCl}_\theta(f^{-1}(V)) \subset f^{-1}(\mathrm{Cl}(V))$ *for every open set V of Y;*

(4) $f^{-1}(V) \subset \mathrm{mInt}_\theta(f^{-1}(\mathrm{Cl}(V)))$ *for every open set V of Y;*

(5) $f(\mathrm{mCl}_\theta(A)) \subset \mathrm{Cl}_\theta(f(A))$ *for every subset A of X;*

(6) $\mathrm{mCl}_\theta(f^{-1}(\mathrm{Int}(\mathrm{Cl}_\theta(B)))) \subset f^{-1}(\mathrm{Cl}_\theta(B))$ *for every subset B of Y;*

(7) $\mathrm{mCl}_\theta(f^{-1}(\mathrm{Int}(\mathrm{Cl}(V)))) \subset f^{-1}(\mathrm{Cl}(V))$ *for every open set V of Y;*

(8) $\mathrm{mCl}_\theta(f^{-1}(\mathrm{Int}(R))) \subset f^{-1}(R)$ *for every regular closed set R of Y;*

(9) $\mathrm{mCl}_\theta(f^{-1}(\mathrm{Int}(K))) \subset f^{-1}(K)$ *for every closed set K of Y.*

Proof. (1) $\Rightarrow$ (2): Let B be any subset of Y. Suppose that $x \notin f^{-1}(\mathrm{Cl}_\theta(B))$. Then $x \in f^{-1}(Y - \mathrm{Cl}_\theta(B)) = f^{-1}(\mathrm{Int}_\theta(Y - B))$. Hence we have $f(x) \in \mathrm{Int}_\theta(Y - B)$. There exists an open set V such that $f(x) \in V \subset \mathrm{Cl}(V) \subset Y - B$. Since f is θ-m-continuous, there exists $U \in m_X$ containing x such that $f(\mathrm{mCl}(U)) \subset \mathrm{Cl}(V)$; hence $\mathrm{mCl}(U) \subset f^{-1}(\mathrm{Cl}(V)) \subset f^{-1}(Y - B) = X - f^{-1}(B)$. Thus we have $\mathrm{mCl}(U) \cap f^{-1}(B) = \emptyset$ and hence $x \notin \mathrm{mCl}_\theta(f^{-1}(B))$.

(2) $\Rightarrow$ (3): This is obvious since $\mathrm{Cl}(V) = \mathrm{Cl}_\theta(V)$ for every open set V of Y.

(3) $\Rightarrow$ (4): Let V be any open set of Y. Then by Lemma 3.3 we have

$$X - \mathrm{mInt}_\theta(f^{-1}(\mathrm{Cl}(V))) = \mathrm{mCl}_\theta(X - f^{-1}(\mathrm{Cl}(V))) = \mathrm{mCl}_\theta(f^{-1}(Y - \mathrm{Cl}(V))) \subset f^{-1}(\mathrm{Cl}(Y - \mathrm{Cl}(V))) \subset f^{-1}(\mathrm{Cl}(Y - V)) = f^{-1}(Y - V) = X - f^{-1}(V).$$

Therefore, we obtain $f^{-1}(V) \subset \mathrm{mInt}_\theta(f^{-1}(\mathrm{Cl}(V)))$.

(4) $\Rightarrow$ (1): Let $x \in X$ and V be any open set of Y containing $f(x)$. Then $x \in f^{-1}(V) \subset \mathrm{mInt}_\theta(f^{-1}(\mathrm{Cl}(V)))$. Therefore, there exists $U \in m_X$ containing x such that $\mathrm{mCl}(U) \subset f^{-1}(\mathrm{Cl}(V))$; hence $f(\mathrm{mCl}(U)) \subset \mathrm{Cl}(V)$. This shows that f is θ-m-continuous.

(2) $\Rightarrow$ (5): Let A be any subset of X. By (2), we have $\mathrm{mCl}_\theta(A) \subset \mathrm{mCl}_\theta(f^{-1}(f(A))) \subset f^{-1}(\mathrm{Cl}_\theta(f(A)))$. Thus, we obtain $f(\mathrm{mCl}_\theta(A)) \subset \mathrm{Cl}_\theta(f(A))$.

(5) $\Rightarrow$ (2): Let B be any subset of Y. Then by (5), we have $f(\mathrm{mCl}_\theta(f^{-1}(B)) \subset \mathrm{Cl}_\theta(f(f^{-1}(B))) \subset \mathrm{Cl}_\theta(B)$. Thus we obtain $\mathrm{mCl}_\theta(f^{-1}(B)) \subset f^{-1}(\mathrm{Cl}_\theta(B))$.

(3) $\Rightarrow$ (6): Let B be any subset of Y. Since $\mathrm{Cl}_\theta(B)$ is closed in Y, by (3) we have $\mathrm{mCl}_\theta(f^{-1}(\mathrm{Int}(\mathrm{Cl}_\theta(B)))) \subset f^{-1}(\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}_\theta(B)))) \subset f^{-1}(\mathrm{Cl}_\theta(B))$.

(6) $\Rightarrow$ (7): This is obvious since $\mathrm{Cl}(V) = \mathrm{Cl}_\theta(V)$ for every open set V of Y.

(7) $\Rightarrow$ (8): Let R be any regular closed set of Y. Then by (7) we have $\mathrm{mCl}_\theta(f^{-1}(\mathrm{Int}(R)))$ $= \mathrm{mCl}_\theta(f^{-1}(\mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(R))))) \subset f^{-1}(\mathrm{Cl}(\mathrm{Int}(R))) = f^{-1}(R)$.

(8) $\Rightarrow$ (9): Let K be any closed set of Y. Since $\mathrm{Cl}(\mathrm{Int}(K))$ is regular closed, we have $\mathrm{mCl}_\theta(f^{-1}(\mathrm{Int}(K))) = \mathrm{mCl}_\theta(f^{-1}(\mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(K))))) \subset f^{-1}(\mathrm{Cl}(\mathrm{Int}(K))) \subset f^{-1}(K)$.

(9) $\Rightarrow$ (4): Let V be any open set of Y. Then $Y - V$ is closed in Y and by (9) we have $\mathrm{mCl}_\theta(f^{-1}(\mathrm{Int}(Y - V))) \subset f^{-1}(Y - V) = X - f^{-1}(V)$. Moreover, we have $\mathrm{mCl}_\theta(f^{-1}(\mathrm{Int}(Y - V))) = \mathrm{mCl}_\theta(f^{-1}(Y - \mathrm{Cl}(V))) = \mathrm{mCl}_\theta(X - f^{-1}(\mathrm{Cl}(V))) = X - \mathrm{mInt}_\theta(f^{-1}(\mathrm{Cl}(V)))$. Therefore, we obtain $f^{-1}(V) \subset \mathrm{mInt}_\theta(f^{-1}(\mathrm{Cl}(V)))$.

Remark 3.5. Let $f : (X, \tau) \to (Y, \sigma)$ be a function. If $m_X = \tau$ (resp. $\mathrm{SO}(X)$, $\mathrm{PO}(X)$) and $f : (X, m_X) \to (Y, \sigma)$ is θ-m-continuous, then by Theorem 3.1 we obtain the characterizations established in Theorem 5 of [9], Theorem 2 of [15], Theorem 5.3 of [18], Theorem 2 of [37] (resp. Theorem 3.1 of [27], Theorems 3.1 and 3.2 of [28], Theorem 3.2 of [7]).

Theorem 3.2. *For a function $f : (X, m_X) \to (Y, \sigma)$, the following properties are equivalent:*

(1) *f is θ-m-continuous;*
(2) $\mathrm{mCl}_\theta(f^{-1}(\mathrm{Int}(\mathrm{Cl}(G)))) \subset f^{-1}(\mathrm{Cl}(G))$ *for every $G \in \beta(Y)$;*
(3) $\mathrm{mCl}_\theta(f^{-1}(\mathrm{Int}(\mathrm{Cl}(G)))) \subset f^{-1}(\mathrm{Cl}(G))$ *for every $G \in \mathrm{SO}(Y)$.*

Proof. (1) $\Rightarrow$ (2): Let $G \in \beta(Y)$. Then $G \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(G)))$ and $\mathrm{Cl}(G) = \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(G)))$. Since $\mathrm{Cl}(G)$ is a regular closed set, by Theorem 3.1 we have $\mathrm{mCl}_\theta(f^{-1}(\mathrm{Int}(\mathrm{Cl}(G))) \subset f^{-1}(\mathrm{Cl}(G))$.

(2) $\Rightarrow$ (3): This is obvious since $\mathrm{SO}(X) \subset \beta(Y)$.

(3) $\Rightarrow$ (1): Let G be any open set. Since $\mathrm{Cl}(G)$ is regular closed, $\mathrm{Cl}(G) \in \mathrm{SO}(Y)$. Thus we have $\mathrm{mCl}_\theta(f^{-1}(\mathrm{Int}(\mathrm{Cl}(G))) \subset f^{-1}(\mathrm{Cl}(G))$. By Theorem 3.1, f is θ-m-continuous.

Theorem 3.3. *For a function $f : (X, m_X) \to (Y, \sigma)$, the following properties are equivalent:*

(1) *f is θ-m-continuous;*
(2) $\mathrm{mCl}_\theta(f^{-1}(\mathrm{Int}(\mathrm{Cl}(G)))) \subset f^{-1}(\mathrm{Cl}(G))$ *for every $G \in \mathrm{PO}(Y)$;*
(3) $\mathrm{mCl}_\theta(f^{-1}(G)) \subset f^{-1}(\mathrm{Cl}(G))$ *for every $G \in \mathrm{PO}(Y)$;*
(4) $f^{-1}(G) \subset \mathrm{mInt}_\theta(f^{-1}(\mathrm{Cl}(G)))$ *for every $G \in \mathrm{PO}(Y)$.*

Proof. (1) $\Rightarrow$ (2): It follows from Theorem 3.2 since $\mathrm{PO}(X) \subset \beta(Y)$.

(2) ⇒ (3): Let G be a preopen set of Y. Then we have

$$\mathrm{mCl}_\theta(f^{-1}(G)) \subset \mathrm{mCl}_\theta(f^{-1}(\mathrm{Int}(\mathrm{Cl}(G)))) \subset f^{-1}(\mathrm{Cl}(G)).$$

(3) ⇒ (4): Let G be any preopen set of Y. By Lemma 3.3, we have

$$X - \mathrm{mInt}_\theta(f^{-1}(\mathrm{Cl}(G))) = \mathrm{mCl}_\theta(X - f^{-1}(\mathrm{Cl}(G))) = \mathrm{mCl}_\theta(f^{-1}(Y - \mathrm{Cl}(G))) \subset f^{-1}(\mathrm{Cl}(Y - \mathrm{Cl}(G))) = X - f^{-1}(\mathrm{Int}(\mathrm{Cl}(G))) \subset X - f^{-1}(G).$$

Therefore, we obtain $f^{-1}(G) \subset \mathrm{mInt}_\theta(f^{-1}(\mathrm{Cl}(G)))$.

(4) ⇒ (1): Let G be any open set of Y. Then we have $G \in \mathrm{PO}(X)$ and hence $f^{-1}(G) \subset \mathrm{mInt}_\theta(f^{-1}(\mathrm{Cl}(G)))$. By Theorem 3.1, f is θ-m-continuous.

Remark 3.6. By Theorems 3.2 and 3.3, we obtain new characterizations for θ-continuity, θ-semi-continuity and θ-precontinuity. For example, for θ-semi-continuous functions we obtain the following corollaries.

Corollary 3.1. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) *f is θ-semi-continuous;*
(2) $\mathrm{sCl}_\theta(f^{-1}(\mathrm{Int}(\mathrm{Cl}(G)))) \subset f^{-1}(\mathrm{Cl}(G))$ *for every $G \in \beta(Y)$;*
(3) $\mathrm{sCl}_\theta(f^{-1}(\mathrm{Int}(\mathrm{Cl}(G)))) \subset f^{-1}(\mathrm{Cl}(G))$ *for every $G \in \mathrm{SO}(Y)$.*

Corollary 3.2. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) *f is θ-semi-continuous;*
(2) $\mathrm{sCl}_\theta(f^{-1}(\mathrm{Int}(\mathrm{Cl}(G)))) \subset f^{-1}(\mathrm{Cl}(G))$ *for every $G \in \mathrm{PO}(Y)$;*
(3) $\mathrm{sCl}_\theta(f^{-1}(G)) \subset f^{-1}(\mathrm{Cl}(G))$ *for every $G \in \mathrm{PO}(Y)$;*
(4) $f^{-1}(G) \subset \mathrm{sInt}_\theta(f^{-1}(\mathrm{Cl}(G)))$ *for every $G \in \mathrm{PO}(Y)$.*

4 θ-m-Continuity and Other Forms of m-Continuity

Lemma 4.1. (Popa and Noiri [41]) *For a function $f : (X, m_X) \to (Y, \sigma)$, the following properties are equivalent:*

(1) *f is almost m-continuous;*
(2) $f^{-1}(V) = \mathrm{mInt}(f^{-1}(V))$ *for every regular open set V of Y;*
(3) $f^{-1}(K) = \mathrm{mCl}(f^{-1}(K))$ *for every regular closed set K of Y.*

Theorem 4.1. *For a function $f : (X, m_X) \to (Y, \sigma)$, the following properties hold:*

(1) *if f is almost m-continuous, then it is θ-m-continuous,*
(2) *if f is θ-m-continuous, then it is weakly m-continuous.*

Proof. (1) Let $x \in X$ and V be any open set of Y containing $f(x)$. Since f is almost m-continuous, there exists $U \in m_X$ containing x such that $f(U) \subset \mathrm{Int}(\mathrm{Cl}(V)) \subset \mathrm{Cl}(V)$. Hence we have $U \subset f^{-1}(\mathrm{Cl}(V))$. Since $\mathrm{Cl}(V)$ is regular closed, by Lemma 4.1 $f^{-1}(\mathrm{Cl}(V)) = \mathrm{mCl}(f^{-1}(\mathrm{Cl}(V)))$. Therefore, we have $\mathrm{mCl}(U) \subset \mathrm{mCl}(f^{-1}(\mathrm{Cl}(V))) = f^{-1}(\mathrm{Cl}(V))$. Hence we obtain $f(\mathrm{mCl}(U)) \subset \mathrm{Cl}(V)$. This shows that f is θ-m-continuous.

(2) This is obvious.

Remark 4.1. Let $f : (X, \tau) \to (Y, \sigma)$ be a function. If $m_X = \tau$ (resp. $\mathrm{SO}(X)$, $\mathrm{PO}(X)$) and $f : (X, m_X) \to (Y, \sigma)$ is almost m-continuous, then by Theorem 4.1 we obtain Theorem of [35] (resp. Theorem 2.4 of [6], Theorem 3.3 of [28]).

Definition 4.1. A topological space (X, τ) is said to be *almost-regular* [46] if for each regular-closed set F and each $x \notin F$, there exist disjoint open sets U and V such that $x \in U$ and $F \subset V$.

Let (X, τ) be a topological space. Since the intersection of two clopen sets of (X, τ) is clopen, the family of clopen sets of (X, τ) may be used as a base for a topology for X. This topology is called the *ultra-regularization* of τ [29] and is denoted by τ_u. A topological space (X, τ) is said to be *ultra-regular* [14] if $\tau = \tau_u$.

Definition 4.2. A function $f : (X, m_X) \to (Y, \sigma)$ is said to be *faintly m-continuous* [31] (resp. *slightly m-continuous* [39]) if for each $x \in X$ and each θ-open (resp. clopen) set V of Y containing $f(x)$, there exists $U \in m_X$ containing x such that $f(U) \subset V$.

Lemma 4.2. (Noiri and Popa [31], Popa and Noiri [39]) *For a function $f : (X, m_X) \to (Y, \sigma)$, the following properties hold:*

(1) *weak m-continuity implies faint m-continuity and faint m-continuity implies slight m-continuuity,*

(2) *if f is faintly continuous and Y is almost-regular, it is almost m-continuous,*

(3) *if f is faintly continuous and Y is regular, it is m-continuous,*

(4) *if f is slightly continuous and Y is ultra-regular, it is m-continuous.*

Corollary 4.1. *For a function $f : (X, m_X) \to (Y, \sigma)$, the following properties hold:*

(1) *m-C $\Rightarrow$ almost m-C $\Rightarrow$ θ-m-C $\Rightarrow$ weak m-C $\Rightarrow$ faint m-C $\Rightarrow$ slight m-C,*

(2) *if Y is almost-regular, then almost m-C, θ-m-C, weak m-C and faint m-C are all equivalent,*

(3) *if Y is regular, then m-C, almost m-C, θ-m-C, weak m-C and faint m-C are all equivalent,*

(4) *if Y is ultra-regular, then m-C, almost m-C, θ-m-C, weak m-C, faint m-C and slight m-C are all equivalent, where C denotes continuity.*

Proof. This is an immediate consequence of Theorem 4.1 and Lemma 4.2.

5 Properties of θ-m-Continuous Functions

Theorem 5.1. *If $f : (X, m_X) \to (Y, \sigma)$ is θ-m-continuous, then the following properties hold:*

(1) *the inverse image of every θ-closed set of Y is m-θ-closed,*

(2) *the inverse image of every θ-open set of Y is m-θ-open.*

Proof. (1) Let B be any θ-closed set of Y. Then $\mathrm{Cl}_\theta(B) = B$. Since f is θ-m-continuous, by Theorem 3.1 we have $\mathrm{mCl}_\theta(f^{-1}(B)) \subset f^{-1}(\mathrm{Cl}_\theta(B)) = f^{-1}(B)$. Hence $\mathrm{mCl}_\theta(f^{-1}(B)) = f^{-1}(B)$. This shows that $f^{-1}(B)$ is m-θ-closed.

(2) Let V be any θ-open set of Y. Then $Y - V$ is θ-closed and $X - f^{-1}(V) = f^{-1}(Y - V)$ is m-θ-closed. Hence $f^{-1}(V)$ is m-θ-open.

Remark 5.1. If $f : (X, \tau) \to (Y, \sigma)$ is a θ-continuous function, then by Theorem 5.1 we obtain the results established in Corollary 5.4 of [18], Theorem 3 of [37] and Lemma 3.2 of [45].

Theorem 5.2. *Let $f : (X, m_X) \to (Y, \sigma)$ be a function. If $f^{-1}(\mathrm{Cl}_\theta(B))$ is m-θ-closed in X for every subset B of Y, then f is θ-m-continuous.*

Proof. Let B be a subset of Y. Since $f^{-1}(\mathrm{Cl}_\theta(B))$ is m-θ-closed , $\mathrm{mCl}_\theta(f^{-1}(\mathrm{Cl}_\theta(B))) = f^{-1}(\mathrm{Cl}_\theta(B))$. Then we have $\mathrm{mCl}_\theta(f^{-1}(B)) \subset \mathrm{mCl}_\theta(f^{-1}(\mathrm{Cl}_\theta(B))) = f^{-1}(\mathrm{Cl}_\theta(B))$. Therefore, we obtain $\mathrm{mCl}_\theta(f^{-1}(B)) \subset f^{-1}(\mathrm{Cl}_\theta(B))$ for every subset B of Y. Then by Theorem 3.1, f is θ-m-continuous.

Corollary 5.1. (Popa [37]) *Let $f : (X, \tau) \to (Y, \sigma)$ be a function. If $f^{-1}(\mathrm{Cl}_\theta(B))$ is θ-closed in X for every subset B of Y, then f is θ-continuous.*

Definition 5.1. Let (X, m_X) be an m-space. A subset K of X is said to be *m-closed relative to* (X, m_X) [30] if for any cover $\{V_\alpha : \alpha \in \Delta\}$ of K by m_X-open sets of (X, m_X), there exists a finite subset Δ_0 of Δ such that $K \subset \bigcup\{\mathrm{mCl}(V_\alpha) : \alpha \in \Delta_0\}$. If X is m-closed relative to (X, m_X), then (X, m_X) is said to be *m-closed*.

Remark 5.2. Let (X, τ) be a topological space and $m = \tau$ (resp. $\mathrm{SO}(X)$, $\mathrm{PO}(X)$, $\delta\mathrm{PO}(X)$). The definition of "m-closed" gives the one of *quasi H-closed* [42] (resp. *s-closed* [11], *p-closed* [12], *δ_p-closed* [44]).

Theorem 5.3. *If $f : (X, m_X) \to (Y, \sigma)$ is a θ-m-continuous function from an m-closed m-space (X, m_X) onto a Urysohn space (Y, σ), then f is almost m-continuous.*

Proof. First, we shall show that (Y, σ) is quasi-H-closed. Let $\{V_\alpha : \alpha \in \Delta\}$ be any open cover of Y. For each $x \in X$, there exists $\alpha(x) \in \Delta$ such that $f(x) \in V_{\alpha(x)}$. Since f is θ-m-continuous, there exists an m_X-open set $U(x)$ containing x such that $f(\mathrm{mCl}(U(x))) \subset \mathrm{Cl}(V_{\alpha(x)})$. The family $\{U(x) : x \in K\}$ is a cover of X by m_X-open sets of X. Since (X, m_X) is m-closed, there exist a finite number of points, say, $x_1, x_2, ..., x_n$ in X such that $X \subset \bigcup\{\mathrm{mCl}(U(x_k)) : x_k \in X, 1 \le k \le n\}$. Therefore, we obtain

$$Y = f(X) \subset \bigcup\{f(\mathrm{mCl}(U(x_k))) : x_k \in X, 1 \le k \le n\}$$
$$\subset \bigcup\{\mathrm{Cl}(V_{\alpha(x_k)}) : x_k \in X, 1 \le k \le n\}.$$

This shows that (Y, σ) is quasi-H-closed. Every quasi-H-closed Urysohn space is almost-regular [34]. By Corollary 4.1, f is almost m-continuous.

Remark 5.3. Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then by Theorem 5.3 we obtain the result established in Theorem 5 of [25].

6 New Forms of θ-m-Continuity

First we recall the relationships among some generalizations of open sets. If a subset A of a topological space (X, τ) is semi-open and semi-closed, then it is said to be *semi-regular* [11]. It is shown in [11] that the semi-closure $\mathrm{sCl}(U)$ is semi-open and semi-regular for any semi-open set U of (X, τ). This property is very useful. The set of all semi-regular sets of (X, τ) is denoted by $\mathrm{SR}(X)$. For a subset A of a topological space (X, τ), we put $\mathrm{srCl}(A) = \cap\{F : A \subset F, F \in \mathrm{SR}(X)\}$.

Let A be a subset of a topological space (X, τ). A point x of X is called a *semi-θ-cluster point* of A if $\mathrm{sCl}(U) \cap A \neq \emptyset$ for every $U \in \mathrm{SO}(X)$ containing x. The set of all semi-θ-cluster points of A is called the *semi-θ-closure* [11] of A and is denoted by $\mathrm{sCl}_\theta(A)$. A subset A is said to be *semi-θ-closed* if $A = \mathrm{sCl}_\theta(A)$. The complement of a semi-θ-closed set is said to be *semi-θ-open*. The family of all semi-θ-open sets of (X, τ) is denoted by $\theta\mathrm{SO}(X)$. A subset A of a topological space (X, τ) is said to be *b-open* [5] if $A \subset \mathrm{Cl}(\mathrm{Int}(A)) \cup \mathrm{Int}(\mathrm{Cl}(A))$.

We have the following diagram in which the converses of implications need not be true as shown in [32].

DIAGRAM I

regular open	→	δ-open	→	open	→	α-open	→	preopen	→	δ-preopen
↓		↓		|		↓		↓		|
semi-regular	→	semi-θ-open	→	δ-semi-open	→	semi-open	→	b-open	→	semi-preopen

Remark 6.1. In the diagram above, the following are to be noted:

(1) It is shown in [36] that openness and δ-semi-openness are independent of each other,

(2) It is shown in [32] that δ-preopenness and semi-preopenness are independent of each other.

If we take as m_X the families of generalized open sets stated in the diagram, we can define new θ-m-continuous functions. By the results established in Sections 3-5, we can obtain those properties. We investigate the relationships among these functions.

Lemma 6.1. *Let m_1 and m_2 be two m-structures on a nonempty set X. If $m_1 \subset m_2$ and a function $f : (X, m_1) \to (Y, \sigma)$ is θ-m-continuous, then $f : (X, m_2) \to (Y, \sigma)$ is θ-m-continuous.*

Proof. Let $x \in X$ and V be any open set of Y containing $f(x)$. Since $f : (X, m_1) \to (Y, \sigma)$ is θ-m-continuous, there exists $U \in m_1$ containing x such that $f(\mathrm{m}_1\,\mathrm{Cl}(U)) \subset \mathrm{Cl}(V)$. Since $m_1 \subset m_2$, we have $x \in U \in m_2$ and $\mathrm{m}_2\,\mathrm{Cl}(U) \subset \mathrm{m}_1\,\mathrm{Cl}(U)$. Therefore, we obtain $f(\mathrm{m}_2\,\mathrm{Cl}(U)) \subset \mathrm{Cl}(V)$. This shows that $f : (X, m_2) \to (Y, \sigma)$ is θ-m-continuous.

Let $\mathrm{RO}(X)$ (resp. $\mathrm{RC}(X)$) be the family of all regular open (resp. regular closed) sets of a topological space (X, τ). The family of all δ-open sets of (X, τ) forms a topology for X which is weaker than τ. This topology has $\mathrm{RO}(X)$ as the base. We shall denote it by τ_δ although it is usually denoted by τ_s. Then we have $\mathrm{RO}(X) \subset \tau_\delta \subset \tau \subset \tau^\alpha$, where $\tau^\alpha = \alpha(X)$. For a subset A of X, we set $\mathrm{rCl}(A) = \cap\{K : A \subset K \text{ and } K \in \mathrm{RC}(X)\}$.

Lemma 6.2. *Let (X, τ) be a topological space. Then $\alpha\mathrm{Cl}(U) = \mathrm{rCl}(\mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(U))))$ for every $U \in \alpha(X)$.*

Proof. Let U be any α-open set of (X, τ). Since $\mathrm{RO}(X) \subset \tau \subset \tau^\alpha$, we have $\alpha\mathrm{Cl}(U) \subset \mathrm{Cl}(U) \subset \mathrm{rCl}(U)$. Suppose that $x \notin \alpha\mathrm{Cl}(U)$. There exists $G \in \tau^\alpha$ containing x such that $G \cap U = \emptyset$. Hence we have $\mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(G))) \cap U \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(G))) \cap \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(U))) = \emptyset$. Since $x \in G \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(G))) \in \mathrm{RO}(X)$, we have $x \notin \mathrm{rCl}(U)$. Therefore, we obtain $\mathrm{rCl}(U) \subset \alpha\mathrm{Cl}(U)$ and $\alpha\mathrm{Cl}(U) = \mathrm{Cl}(U) = \mathrm{rCl}(U)$ for every $U \in \alpha(X)$. Moreover, for every $U \in \alpha(X)$, we have $\mathrm{Cl}(U) = \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(U)))) = \mathrm{rCl}(\mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(U))))$. Therefore, we obtain $\alpha\mathrm{Cl}(U) = \mathrm{rCl}(\mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(U))))$ for every $U \in \alpha(X)$.

Theorem 6.1. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) $f : (X, \mathrm{RO}(X)) \to (Y, \sigma)$ *is θ-m-continuous;*
(2) $f : (X, \tau_\delta) \to (Y, \sigma)$ *is θ-m-continuous;*
(3) $f : (X, \tau) \to (Y, \sigma)$ *is θ-m-continuous;*
(4) $f : (X, \tau^\alpha) \to (Y, \sigma)$ *is θ-m-continuous.*

Proof. Since $\mathrm{RO}(X) \subset \tau_\delta \subset \tau \subset \tau^\alpha$, by Lemma 6.1 we have (1) $\Rightarrow$ (2) $\Rightarrow$ (3) $\Rightarrow$ (4).

(4) $\Rightarrow$ (1): Let $x \in X$ and V be any open set of Y containing $f(x)$. There exists an α-open set U containing x such that $f(\alpha\mathrm{Cl}(U)) \subset \mathrm{Cl}(V)$. Since $U \in \tau^\alpha$, we have $x \in U \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(U))) \in \mathrm{RO}(X)$. By Lemma 6.2, we have $f(\mathrm{rCl}(\mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(U))))) = f(\alpha\mathrm{Cl}(U)) \subset \mathrm{Cl}(V)$. This shows that $f : (X, \mathrm{RO}(X)) \to (Y, \sigma)$ is θ-m-continuous.

Corollary 6.1. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) $f : (X, \tau) \to (Y, \sigma)$ *is θ-continuous;*
(2) $f : (X, \tau_\delta) \to (Y, \sigma)$ *is θ-continuous;*
(3) $f : (X, \tau^\alpha) \to (Y, \sigma)$ *is θ-continuous.*

Theorem 6.2. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) $f : (X, \mathrm{SR}(X)) \to (Y, \sigma)$ *is θ-m-continuous;*
(2) $f : (X, \theta\mathrm{SO}(X)) \to (Y, \sigma)$ *is θ-m-continuous;*
(3) $f : (X, \delta\mathrm{SO}(X)) \to (Y, \sigma)$ *is θ-m-continuous;*
(4) $f : (X, \mathrm{SO}(X)) \to (Y, \sigma)$ *is θ-m-continuous.*

Proof. Since $\mathrm{SR}(X) \subset \theta\mathrm{SO}(X) \subset \delta\mathrm{SO}(X) \subset \mathrm{SO}(X)$, by Lemma 6.1 we have (1) $\Rightarrow$ (2) $\Rightarrow$ (3) $\Rightarrow$ (4).

(4) $\Rightarrow$ (1): Suppose that $f : (X, \mathrm{SO}(X)) \to (Y, \sigma)$ is θ-m-continuous. Let $x \in X$ and V be any open set of Y containing $f(x)$. There exists $U \in \mathrm{SO}(X)$ containing x such that $f(\mathrm{sCl}(U)) \subset \mathrm{Cl}(V)$. By Proposition 2.2 of [11], $\mathrm{sCl}(U) \in \mathrm{SR}(X)$ and we have $x \in \mathrm{sCl}(U)$. Moreover, we have $\mathrm{srCl}(\mathrm{sCl}(U)) = \mathrm{sCl}(U)$. Therefore, we obtain $f(\mathrm{srCl}(\mathrm{sCl}(U))) = f(\mathrm{sCl}(U)) \subset \mathrm{Cl}(V)$. This shows that $f : (X, \mathrm{SR}(X)) \to (Y, \sigma)$ is θ-m-continuous.

Remark 6.2. (1) In [6], it is shown that every θ-continuous function is θ-semi-continuous but the converse is not correct.

(2) Every θ-continuous function is θ-precontinuous. However, by Example 4.2 of [7], it turns out that the converse is not always true.

References

[1] Abd El-Monsef, M.E., El-Deeb, S.N. and Mahmoud, R.A., *β-open sets and β-continuous mappings*, Bull. Fac. Sci. Assiut Univ. **12** (1983), 77–90.

[2] Abd El-Monsef, M.E., Mahmoud, R.A. and Lashin, E.R., *β-closure and β-interior*, J. Fac. Ed. Ain Shams Univ. **10** (1986), 235–245.

[3] Andrew, D.R. and Whittlesy, E.K., *Closure continuity*, Amer. Math. Monthly **73** (1966), 758–759.

[4] Andrijević, D., *Semi-preopen sets*, Mat. Vesnik **38** (1986), 24–32.

[5] Andrijević, D., *On b-open sets*, Mat. Vesnik **48** (1986), 50–64.

[6] Arya, S.P. and Bhamini, M.P., *Some weaker forms of semi-continuous functions*, Ganita **33** (1982), 124–134.

[7] Baker, C.W., *Weakly θ-precontinuous functions*, Acta Math. Hungar. **100** (2003), 343–351.

[8] Chew, J., *A note on closure continuity*, Amer. Math. Monthly **95** (1988), 744–745.

[9] Chew, J., *More remarks on generalized forms of continuity*, Anal. Univ. Craiova, Ser. Mat. **19** (1991/92), 3–8.

[10] Crossley, S.G. and Hildebrand, S.K., *Semi-closure*, Texas J. Sci. **22** (1971), 99–112.

[11] Di Maio, G. and Noiri, T., *On s-closed spaces*, Indian J. Pure Appl. Math. **18** (1987), 226–233.

[12] Dontchev, J., Ganster, M. and Noiri, T., *On p-closed spaces*, Internat. J. Math. Math. Sci. **24** (2000), 203–212.

[13] El-Deeb, S.N., Hasanein, I.A., Mashhour, A.S. and Noiri, T., *On p-regular spaces*, Bull. Math. Soc. Sci. Math. R. S. Roumanie **27(75)** (1983), 311–315.

[14] Elis, R.E., *A nonarchimedian analog of the Tietze-Urysohn extension theorem*, Indag. Math. **70** (1967), 332–333.

[15] Espelie, M.S. and Joseph, J.E., *Remarks on two weak forms of continuity*, Canad. Math. Bull. **25** (1982), 59–63.

[16] Fomin, S., *Extensions of topological spaces*, Ann. of Math. **44** (1943), 471–480 = Dokl. Akad. Nauk. S. S. S. R. **32** (1941), 114–116.

[17] Jafari, S. and Noiri, T., *Properties of θ-semi-continuous functions*, J. Inst. Math. Comput. Sci. Math. Ser. **13** (2000), 123–128.

[18] Janković, D.S., *On some separation axioms and θ-closure*, Mat. Vesnik. **4(17)(32)** (1980), 439–449.

[19] Janković, D.S., *On functions with θ-closed graphs*, Rend. Math. Appl. **7** (1983), 73–78.

[20] Levine, N., *Semi-open sets and semi-continuity in topological spaces,* Amer. Math. Monthly **70** (1963), 36–41.

[21] Maki, H., Chandrasekhara Rao, K. and Nagoor Gani, A., *On generalizing semi-open sets and preopen sets*, Pure Appl. Math. Sci. **49** (1999), 17–29.

[22] Mashhour, A.S., Abd El-Monsef, M.E. and El-Deep, S.N., *On precontinuous and weak precontinuous mappings,* Proc. Math. Phys. Soc. Egypt **53** (1982), 47–53.

[23] Mashhour, A.S., Hasanein, I.A. and El-Deeb, S.N., *α-continuous and α-open mappings*, Acta Math. Hungar. **41** (1983), 213–218.

[24] Njåstad, O., *On some classes of nearly open sets,* Pacific J. Math. **15** (1965), 961–970.

[25] Noiri, T., *Between continuity and weak-continuity,* Boll. Un. Mat. Ital. (4) **9** (1974), 647–654.

[26] Noiri, T., *Properties of θ-continuous functions*, Atti Accad. Naz. Lincei, Rend. Cl. Sci. Fiz. Mat. Natur. Ser. (8) **58** (1975), 887–891.

[27] Noiri, T., *On θ-semi-continuous functions*, Indian J. Pure Appl. Math. **21** (1990), 410–415.

[28] Noiri, T., *On θ-precontinuous functions*, Internat. J. Math. Math. Sci. **28** (2001), 285–292.

[29] Noiri, T., *Weak and strong forms of β-irresolute functions*, Acta Math. Hungar. **99** (2003), 305-318.

[30] Noiri, T. and Popa, V., *A unified theory of θ-continuity for functions*, Rend. Circ. Mat. Palermo (2) **52** (2003), 163–188.

[31] Noiri, T. and Popa, V., *Faintly m-continuous functions*, Chaos, Solitons and Fractals **19** (2004), 1147–1159.

[32] Noiri, T. and Popa, V., *On m-quasi-irresolute functions* (submitted).

[33] Pal, M.C. and Bhattacharyya, P., *Feeble and strong forms of preirresolute functions*, Bull. Malaysian Math. Soc. (2) **19** (1996), 63–75.

[34] Papić, P., *Sur les espaces H-fermés*, Glasnik Mat. Fiz. Astr. **14** (1959), 135–141.

[35] Papp, F.J., *Almost-continuous implies θ-continuous*, Notices Amer. Math. Soc. **20** (1973), P. A-175.

[36] Park, J.H., Lee, B.Y. and Son, M.J., *On δ-semiopen sets in topological spaces,* J. Indian Acad. Math. **19** (1997), 59–67.

[37] Popa, V., *Characterizations of θ-continuous functions*, Stud. Cerc. St. Ser. Mat. (1980), 113–119.

[38] Popa, V. and Noiri, T., *On the definitions of some generalized forms of continuity under minimal conditions*, Mem. Fac. Sci. Kochi Univ. Ser. Math. **22** (2001), 9–18.

[39] Popa, V. and Noiri, T., *Slightly m-continuous functions*, Radovi Math. **10** (2001), 219–232.

[40] Popa, V. and Noiri, T., *On weakly m-continuous functions*, Mathematica (Cluj) **45**(**68**) (2003), 53–67.

[41] Popa, V. and Noiri, T., *On almost m-continuous functions*, Math. Notae **40** (1999-2002), 75–94.

[42] Porter, J. and Thomas, J., *On H-closed and minimal Hausdorff spaces*, Trans. Amer. Math. Soc. **138** (1969), 159–170.

[43] Raychaudhuri, S. and Mukherjee, M.N., *On δ-almost continuity and δ-preopen sets,* Bull. Inst. Math. Acad. Sinica **21** (1993), 357–366.

[44] Raychaudhuri, S. and Mukherjee, M.N., *δ_p-closedness for topological spaces,* J. Indian Acad. Math. **18**(1996), 89–99.

[45] Saleh, M., *On θ-continuity and strong θ-continuity*, Applied Math. E-Notes **3** (2003), 42–48.

[46] Singal, A.R. and Arya, S.P., *On almost regular spaces*, Glasnik Mat. Ser. III (**4**)(**24**) (1969), 89–99.

[47] Veličko, N.V., *H-closed topological spaces,* Amer. Math. Soc. Transl. (2) **78** (1968), 103–118.