LIBERTAS MATHEMATICA, VOL. XIX (1999)

# ON SOME KINDS OF SUBMANIFOLDS OF AN ALMOST HERMITIAN OR PARA- HERMITIAN MANIFOLD

**FERNANDO ETAYO**

Dedicated to Prof. Radu Rosca

**Abstract.** *This is a survey about some kinds of real submanifolds of an almost Hermitian (resp. almost para-Hermitian) manifold, introduced by several authors in the last years.*

**1991 Mathematics Subject Classification:** 53C40, 53C15.

## Introduction

This paper is a survey without proofs about submanifolds of both almost Hermitian and almost para-Hermitian manifolds. We focus on those which are less known or more recent. Thus, we avoid a long explanation about holomorphic, totally real and CR-submanifolds of an almost Hermitian manifold, because they are well known. Our attention will address the folllowing topics: slant submanifolds (resp. other submanifolds) of almost Hermitian manifolds, in section 2 (resp. in section 3); submanifolds of indefinite almost Hermitian manifolds in section 4; in the last section, submanifolds of almost para-Hermitian manifolds, which were firstly studied, among others, by Radu Rosca.

## 1. BASIC DEFINITIONS

We use the following notation, through the first two sections of the paper: $(\overline{M}, \overline{J}, \overline{g})$ is an almost Hermitian manifold and $M$ is a real submanifold of $\overline{M}$. Then, the tangent space $T_x\overline{M}$ of $\overline{M}$ at $x \in M$ may be decomposed as the direct and normal sum $T_x\overline{M} = T_xM \oplus T_x^{\perp}M$, where $T_xM$ (resp. $T_x^{\perp}M$) is the tangent space (resp. the normal space) of $M$ at $x$. As is well known, many of geometric properties of the submanifold $M$ arise from that decomposition, such as Gauss and Weingarten formulas, the induced connection on the submanifold, the relationship between the curvature of the manifold and that of the submanifold, etc...

On the other hand, taking into account the almost complex structure $\overline{J}$ of $\overline{M}$ one can define some classes of submanifolds:

**Definition 1** *$M$ is an* almost complex (or holomorphic) submanifold *of $(\overline{M}, \overline{J}, \overline{g})$ if $\overline{J}(T_xM) = T_xM$, for all $x \in M$.*

**Definition 2** *$M$ is a* totally real (or anti-invariant) submanifold *of $(\overline{M},\overline{J},\overline{g})$ if $\overline{J}(T_xM)\subset T_x^{\perp}M$, for all $x\in M$.*

Almost complex and totally real submanifolds have exhaustively been studied (see e.g. [29] and [38] ).

In 1978 Bejancu introduced (see[5] ) a new class of submanifolds, containing the both above ones:

**Definition 3** *$M$ is a* Cauchy-Riemann submanifold *(or* CR- submanifold*) of $(\overline{M},\overline{J},\overline{g})$ if there exists a differentiable distribution $D: x\mapsto D_x$ on $M$ satisfying the following conditions:*

*(i) $D$ is holomorphic, i.e., $\overline{J}(D_x)=D_x$, for all $x\in M$.*

*(ii) the complementary orthogonal distribution $D^{\perp}: x\mapsto D_x^{\perp}\subset T_xM$ is anti-invariant, i.e., $\overline{J}(D_x^{\perp})\subset T_x^{\perp}M$, for all $x\in M$.*

Obviously, almost complex (resp. totally real) submanifolds are CR- submanifolds having $D_x=T_xM$ (resp. $D_x=0$). One can see [6] for a complete study of this geometric objects.

Another generalization of holomorphic and totally real submanifolds has been presented by B. Y. Chen in the eaerlier 90's (see [13] ):

**Definition 4** *$M$ is a* slant submanifold *of $(\overline{M},\overline{J},\overline{g})$ if the angle between $\overline{J}(X)$ and $T_xM$ is constant for every vector $X\in T_xM$ and every $x\in M$. This angle is called the* Wirtinger angle.

If the Wirtinger angle is $\vartheta\neq 0,\ \frac{\pi}{2}$, the submanifold is said *proper slant*. Almost complex (resp. totally real) submanifolds are slant submanifolds having the Wirtinger angle $\vartheta=0$ (resp. $\vartheta=\frac{\pi}{2}$).

All the submanifolds given in the above definitions verify that the dimension of $\overline{J}(T_xM)\cap T_xM$ is independent on $x\in M$, i.e., the holomorphic spaces $H_x=\overline{J}(T_xM)\cap T_xM$ define a differentiable distribution on $M$. The same author, B. Y. Chen, gave the following name to these manifolds (see e.g. [12] ):

**Definition 5** *$M$ is a* generic submanifold *of $(\overline{M},\overline{J},\overline{g})$ if the holomorphic spaces $H_x=\overline{J}(T_xM)\cap T_xM$ define a differentiable distribution on $M$.*

But then one can easily find submanifolds which are not generic. For example, the canonical embedding of the 2-sphere $S^2\subset R^4=C^2$ given by $\{x_1^2+x_2^2+x_3^2=1; x_4=0\}$ does not give a generic submanifold, because taking $p=(0,0,1,0)$ and $q=(0,1,0,0)$ one observes that $dim\ H_p=0$ whereas $dim\ H_q=2$.

Thus one can wonder what properties has the function $d: x\mapsto dimH_x$ when $x\in M$, $M$ being a real submaifold of $(\overline{M},\overline{J},\overline{g})$. Obviously, $M$ being connected, the following asserts are equivalent: $d$ is continuous; $d$ is constant; $M$ is a generic submanifold.

The general answer is the following:

**Proposition 1** (see [23] ) *Let $M$ be a real submanifold of an almost complex manifold $(\overline{M}, \overline{J})$. The function $d : M \to R$ given by $d(x) = dim : H_x$ is upper- semicontinuous, i.e., the sets $\{x \in M / : d(x) < n\}$ are open subsets of $M$.*

**Example.** Taking into account the above Proposition one can deduce that if $M$ is an n-dimensional real compact submanifold of $C^m$, for some $m$, then the set $\{x \in M / dim\ H_x < n\}$ is a non-empty open subset of $M$, and thus the condition $\overline{J}(T_x M) = T_x M$ fails in more that one point $x \in M$.

## 2. SLANT SUBMANIFOLDS

As we have said in the above section, slant submanifolds have been introduced by Chen in 1990. Now, we are going to point out the main results and topics about this class of submanifolds, which have been developped during this last decade.

As is well known (cfr., e.g., [36] ) a compact maifold cannot be endowed in $C^n$ as a holomorphic submanifold. This result has been generalizaded by Chen and Tazawa:

**Proposition 2** ([16] ) *The only slant immersions of a compact manifold into $C^n$ are the totally real ones.*

Some results about slant surfaces in a 4-dimensional manifolds have been obtained by Chen, Tazawa and Ikawa. We have chosen the following ones:

**Proposition 3** ([15] ) *If $M$ is an oriented surface into $(\overline{M}, \overline{J}, \overline{g})$, where $\overline{M}$ is a 4-dimensional manifold, and there are no points $x \in M$ such that $\overline{J}(T_x M) = T_x M$, then, for any $\vartheta \in (0, \pi)$ there exists an almost Hermitian structure on $(\overline{M}, \overline{g})$ such that $M$ is a slant submanifold with Wirtinger angle equal to $\vartheta$.*

**Proposition 4** ([25] ) *A slant surface with parallel mean curvature in a 4-dimensional flat Kaehler manifold is either minimal or locally a Riemannian product of two circles.*

And we end with this two related results. First, one of Chen:

**Proposition 5** ([14] ) *The helical cylinders in $C^2$ are flat, proper slant surfaces with nonzero mean curvature.*

and second, its converse, proved by Yang in 1997:

**Proposition 6** ([37] ) *A flat, proper slant surface with nonzero constant mean curvature in $C^2$ is an open portion of a helical cylinder.*

As one can see, many works in this topic study geometric properties of slant submanifolds of complex Euclidean spaces $C^m$ (cfr the above quoted papers and [35] ), of complex space forms (cfr. [17] ) and of Kaehler manifolds (cfr. [25, 27] ).

In the next section we will see some generalizations of slant submanifolds.

## 3. OTHER SUBMANIFOLDS

In this section we present several kinds of submanifolds defined in last years. The first definition is a natural generalization of both CR- and slant submanifolds, given by Papaghiuc (cfr. [31] ):

**Definition 6** *The submanifold $M$ is said* semi-slant *if it is endowed with two orthogonal distributions $D$ and $D^{\perp}$, where $D$ is $\overline{J}$-invariant and $D^{\perp}$ is slant, i.e., the angle $\vartheta(X)$ between $\overline{J}(X)$ and $D_x^{\perp}$ is a constant.*

One can easily check that holomorphic and totally real submanifolds are Cauchy-Riemann and slant submanifolds; these are semi-slant submanifolds and all of them are generic submanifolds.

On the other hand, we have introduced (see [20] )

**Definition 7** *A submanifold $M$ of an almost Hermitian manifold $(\overline{M}, \overline{J}, \overline{g})$ is said* **quasi-slant** *if, for each $x \in M$, the angle $\vartheta(X)$ between $\overline{J}(X)$ and $T_xM$ is a constant, for all non-zero vector $X \in T_xM$, i.e., it does not depend on the choice of $X \in T_xM$, but it depends on the choice of the point $x \in M$.*

Then, the angle function can be defined on the submanifold $M$ and it will be denoted as $\vartheta : M \to [0, \frac{\pi}{2}]$, $\vartheta(x) = \vartheta_x$. Obviously, a slant submanifold is a quasi-slant submanifold. Observe that, for each $x \in M$, $M$ being a quasi-slant manifold, the maximal holomorphic subspace $H_x = T_xM \cap \overline{J}(T_xM)$ is $H_x = T_xM$ or $H_x = \{0\}$, thus proving that quasi-slant submanifolds may be no generic submanifolds.

We will give two results about quasi-slant submanifolds. The first one reveals that quasi-slant surfaces are quite different of slant submanifolds (cfr. prop. 2.2-5), because one has:

**Proposition 7** ([20] ) *Let $M$ be a submanifold of an almost Hermitian manifold $(\overline{M}, \overline{J}, \overline{g})$. If $M$ is a surface, then $M$ is a quasi-slant submanifold. Moreover, if $M$ is non- orientable, then there exists a point $x \in M$ such that $\vartheta_x = \frac{\pi}{2}$.*

On the other hand, $M$ being a quasi-slant submanifold of $(\overline{M}, \overline{J}, \overline{g})$, one can define the tensor field $F$ of type (1,1) on $M$ given by $F_x = \pi \circ \overline{\pi} \mid_{T_xM}: T_xM \to T_xM$, where $\pi$ (resp. $\overline{\pi}$) denotes the orthogonal projection over $T_xM$ (resp. $\overline{J}(T_xM)$). If $M$ is a holomorphic (resp. totally real) submanifold, its canonical tensor field is the identity (resp. the null tensor field). Then one obtains:

**Proposition 8** ([20] ) *Let $M$ be a submanifold of an almost Hermitian manifold $(\overline{M}, \overline{J}, \overline{g})$. Then $M$ is a quasi-slant submanifold iff $F_x$ is a homothety, for all $x \in M$.*

Some papers are devoted to the study of submanifolds defined in a way parallel to that of CR-submanifolds. Thus, Mihai gave the following definition in [28] :

**Definition 8** *$M$ is said an* almost CR-submanifold *if the anti- invariant spaces $\{D_x^{\perp} = T_xM \cap \overline{J}(T_x^{\perp}M), x \in M\}$ define a differentiable distribution on $M$.*

## 4. INDEFINITE ALMOST HERMITIAN MANIFOLDS

Indefinite almost Hermitian manifolds were introduced in [3] . The definition is similar to that of positive definite case (i.e., the almost Hermitian manifolds), with the exception of the condition about the metric, which now is semi-Riemannian. Submanifolds of semi-Riemannian manifolds run well if they are also semi-Riemannian manifolds (see, e.g., [30] ), but the geometry of degenerate submanifolds (or lightlike submanifolds) is more difficult (see [10, 26] ). For this reason, this theory has been developped later. The first aim of the theory of lightlike submanifolds consists on obtaining equations of Gauss and Weirgarten kinds. The problem is the non-existence of a decomposition of $T_x\overline{M}$ as $T_xM \oplus T_x^{\perp}M$, because $T_xM \cap T_x^{\perp}M \neq \{0\}$. Then, there exist two ways to obtain such equations: Kupeli's approach (see [26] ) uses a Koszul connection on the submanifold, which exists iff the submanifold verify some conditions, and also needs some quotient tangent bundles. On the other hand, the point of view of Bejancu and Duggal (see [10] ) uses a screen distribution which allows them to obtain $T_x\overline{M}$ as a direct sum of subspaces. Nevertheless, they have to check that the results based on that decomposition does not depend on the chosen screen distribution.

In [10, ch.6] Bejancu and Duggal study CR-lightlike submanifolds of indefinite Kaehler manifolds, following their own papers [7, 8, 9, 19] . If $M$ is a lightlike submanifold of an indefinite almost Hermitian manifold $(\overline{M}, \overline{J}, \overline{g})$, then the metric $g = \overline{g}\mid_M$ is degenerate and then $T_xM \cap T_x^{\perp}M$ is non-trivial when $x \in M$. Thus, definition 1.3 is not possible. Bejancu and Duggal solve this problem by means of the quoted auxiliar screen distribution. The definition is too complicated to be developped here, and one can see the details in their book.

On the other hand, a pointwise classification of submanifolds of an indefinite almost Hermitian manifold of dimension 4 is obtained by Etayo and Fioravanti in [22] , according the possible values of the following three numbers:

$$a_x = \dim\left(T_xM \cap T_x^{\perp}M\right);$$

$$b_x = \text{signature}\left(\overline{g}\mid_M\right)_x;$$

$$c_x = \dim\left(T_xM \cap \overline{J}(T_xM)\right).$$

Then one obtains:

**Proposition 9** ([22] ) *Let $M$ be a submanifold of an indefinite almost Hermitian manifold $(\overline{M}, \overline{J}, \overline{g})$ of dimension 4. Then, the signature of $\overline{g}$ is 4 or 0 and*

*(a) If signature$(\overline{g}) = 4$ at the point $x \in M$, then $M$ belongs to one of the following classes $(dim M, a_x, b_x, c_x)$ at the point $x \in M$:*

$(3,0,3,2), (2,0,2,0), (2,0,2,2), (1,0,1,0)$

*(b) If signature$(\overline{g}) = 0$ at the point $x \in M$, then $M$ belongs to one of the following classes $(dim M, a_x, b_x, c_x)$ at the point $x \in M$:*

$(3,0,1,2),(3,1,0,2)$
$(2,0,0,0),(2,0,2,0),(2,0,2,2),(2,1,1,0),(2,2,0,0),(2,2,0,2)$
$(1,0,1,0),(1,1,0,0)$

## 5. SUBMANIFOLDS OF ALMOST PARA-HERMITIAN MANIFOLDS

$(\overline{M},\overline{J},\overline{g})$ is said an almost para-Hermitian manifold if $\overline{J}$ is a (1,1)-tensor field verifying $\overline{J}^2 = Id$ and $\overline{g}$ is a semi-Riemannian metric such that $\overline{g}(\overline{J}X,\overline{J}Y) = -\overline{g}(X,Y)$ for all tangent vectors to $\overline{M}$. Then, one can easily prove that $\overline{g}$ is a neutral metric, i.e., its signature is $(n,n)$ (see [18] for a global view of this topic). Some relevant aspects of the geometry of paracomplex manifolds are similar to those of the complex ones, and some of them are very different. In general, complex theory has been studied before that paracomplex one, but CR-submanifolds of almost para-Hermitian manifolds were introduced, by Radu Rosca, before CR-submanifolds of indefinite almost Hermitian manifolds.

Now, we point out the main steps in the evolution of the theory of CR- submanifolds of almost para-Hermitian manifolds.

**Definition 9** *$M$ is a* CR-submanifold *of an almost para- Hermitian manifold $(\overline{M},\overline{J},\overline{g})$ if there exist two differentiable distribution $D : x \mapsto D_x$ on $M$ and $D^\perp : x \mapsto D_x^\perp$ satisfying the following conditions:*

*(i) $T_xM = D_x \oplus D_x^\perp$, for all $x \in M$.*
*(ii) $D$ and $D^\perp$ are orthogonal.*
*(iii) $D$ is $\overline{J}$-invariant, i.e., $\overline{J}(D_x) = D_x$, for all $x \in M$.*
*(iv) $D^\perp$ is anti-invariant, i.e., $\overline{J}(D_x^\perp) \subset T_x^\perp M$, for all $x \in M$.*
*(v) The metric $\overline{g}\,|_M$ is of constant signature and rank.*

This definition was introduced by Rosca in [32] . One can observe that it is simpler than that of a CR-submanifold of an indefinite almost Hermitian manifold (see [10] and the comments in the above section). The reason is the following: Bejancu and Duggal, in [10] , want to obtain CR-submanifolds which verify some formulas similar to Gauss and Weigarten equations, while Rosca's point of view is to obtain a theory similar to that of the almost Hermitian case, and he does not look for equations similar to those of Riemannian submanifolds.

Some general results about some kinds of submanifolds of an almost para-Hermitian manifold have been obtained by Rosca [32, 33, 34] , Amato [1, 2] , Etayo and Fioravanti [21] , and others. Some important examples have been studied, such as CR-submanifolds of the paracomplex projective space (see [21] ) or the geometry of the unit tangent bundle as a lightlike hypersurface of the tangent bundle, endowed with the neutral metric (see [24] ).

In [11] Bejancu and Etayo begin the study of lightlike hypersurfaces of an almost para-Hermitian manifold, showing that there exist two classes of such hypersurfaces: *invariant* ones, which obey $\overline{J}(T_x^\perp M) = T_x^\perp M$ and *non-invariant* ones, which verify $\overline{J}(T_x^\perp M) \cap T_x^\perp M = \phi$. Using an auxiliar distribution one can obtain some formulas similar to those of submanifolds of a Riemannian manifold. This study has been continued by Bejan in [4] , where she has obtained geometric properties of 2-codimensional lightlike submanifolds of an almost para-Hermitian manifold. On the other hand, some results relating lightlike hypersurfaces and CR-submanifolds have been obtianed in [21] . In particular it is proved that a lightlike hypersurface is a CR-submanifold iff it is an invariant hypersurface.

All submanifolds of a 4-dimensional almost para-Hermitian manifold have been classified in [24] in a similar way to that given in [22] for the indefinte almost Hermitian case.

**Final remark.**
In this note, we have showed the definitions of some submanifolds of (indefinite) almost Hermitian or para-Hermitian manifolds. The study of submanifolds of manifolds endowed with a geometric structure is continuely growing. Some notions has been succesfully translated to other frameworks (e.g., the theory of CR-submanifolds of a Sasakian manifold) and one can wonder if other notions as the quoted in the present paper will be interesting and useful when they are applied to other geometries.

## REFERENCES


[1] **Amato, F.**: *CR-sous-variétés d'une variété parakaehlerienne possédant la propriété de Poisson.* Rend. Mat. (6) **4** (1984), 351- 355.

[2] **Amato, F.**: *CR-sous-variétés co-isotropes incluses dans une variété pseudo-riemannianne neutre.* Boll. U. M. I. (6) **4-A** (1985), 433-440.

[3] **Barros, M.; Romero, A.**: *Indefinite Kaehler manifolds.* Math. Ann. **261** (1982), 55-62.

[4] **Bejan, C. L.**: *2-codimensional lightlike submanifolds of almost para-Hermitian manifolds.* Differential Geometry and appl., Brno, 1995, 7-17, Masaryk Univ., Brno, 1996.

[5] **Bejancu, A.**: *CR-submanifolds of a Kaehler manifold* **I**. Proc. Amer. Math. Soc. **69** (1978), 134-142.

[6] **Bejancu, A.**: *Geometry of CR-Submanifolds.* D. Reidel, Dordrecht, 1986.

[7] **Bejancu, A.; Duggal, K. L.**: *Degenerate hypersurfaces of semi- Riemannian manifolds.* Bull. Inst. Politehnic Iasi (S.1) **37** (1991), 13-22.

[8] **Bejancu, A.; Duggal, K. L.**: *Real hypersurfaces of indefinite Kaehler manifolds.* Internat. J. Math. **38** (1995), 197-215.

[9] **Bejancu, A.; Duggal, K. L.**: *Lightlike submanifolds of semi- Riemannian manifolds.* Acta Applic. Math. **38** (1995), 197-215.

[10] **Bejancu, A.; Duggal, K. L.**: *Lightlike Submanifolds of semi-Riemannian Manifolds and Applications* Kluwer, Dordrecht, 1996.

[11] **Bejancu. A.; Etayo, F**: *Degenerate Hypersurfaces of Almost-para- Hermitian Manifolds.* J. Tensor Soc. India **12**, 39-56 (1994)

[12] **Chen, B. Y.**: *Geometry of Submanifolds and its Applications.* Sci. Univ. Tokyo, 1981

[13] **Chen, B. Y.**: *Slant immersions.* Bull. Austral. Math. Soc. **41** (1990) , 135-147.

[14] **Chen, B. Y.**: *Geometry of slants submanifolds.* Katholicke Universiteit Leuven, Louvain (1990)

[15] **Chen, B. Y.; Tazawa, Y.**: *Slant surfaces of codimension two.* Ann. Fac. Sci. Toulouse Math. (5) **11** (1990), 29-43.

[16] **Chen, B. Y.; Tazawa, Y.**: *Slant submanifolds in complex Euclidean spaces.* Tokyo J. Math. **14** (1991), 101-120.

[17] **Chen, B. Y.; Vranken, L.**: *Existence and uniqueness theorem for slant immersions and its applications.* Results Math. **31** (1997), 28-39.

[18] **Cruceanu, V.; Fortuny, P.; Gadea, P. M.**: *A survey on Paracomplex Geometry.* Rocky Mountain J. Math. **26** (1996), 83-115.

[19] **Duggal, K. L.; Bejancu, A.**: *Lightlike CR-hypersurfaces of indefnite Kaehler manifolds.* Acta Appl. Math. **31** (1993), 171- 190.

[20] **Etayo, F.**: *On quasi-slant submanifolds of an almost Hermitian manifold.* Publ. Math. Debrecen **53** (1998), 217-223.

[21] **Etayo, F.; Fioravanti, M.**: *CR-submanifolds of the paracomplex projective space.* Publ. Math. Debrecen **47**, 151-160 (1995)

[22] **Etayo, F.; Fioravanti, M.**: *On the Submanifolds of Semi- Riemannian Almost Complex Manifolds.* Atti Sem. Mat. Fis. Univ. Modena **46** (1998), 535-556

[23] **Etayo, F.; Fioravanti, M.; Trịas, U. R.**: *The dimension function of holomorphic spaces of a real submanifold of an almost complex manifold.* To appear in Czech. Math. J.

[24] **Etayo, F.; Fioravanti, M.; Trịas, U. R.**: *On the submanifolds of an almost para-Hermitian manifold.* To appear in Acta Math. Hung.

[25] **Ikawa, T.**: *On the length of the second fundamental form of slant surfaces.* Tensor (N.S.) **52** (1993), 249-254.

[26] **Kupeli, D. N.**: *Singular Semi-Riemannian Geometry,* Kluwer, Dordrecht, 1996.

[27] **Maeda, S.; Ohnita, Y.; Udagawa, S.**: *On slant immersions into Kaehler manifolds.* Kodai Math. J. **16** (1993), 205-219.

[28] **Mihai, I.**: *Certain Submanifolds of a Kaehler Manifold.* Geometry and Topolgy of Submanifolds, VII, (Leuven/Brussels, 1994), 186- 188, World Sci.Publishing, River Edge, N. J., 1995.

[29] **Oguie, K.**: *Differential Geometry of Kaehler Submanifolds.* Advances in Math. **13** (1974), 73-114

[30] **O'Neill, B.**: *Semi-Riemannian Geometry.* Ac. Press, N. York, 1983.

[31] **Papaghiuc, N.**: *Semi-slant submanifolds of a Kaehlerian manifold.* Ann. Stiint. Univ. Al. I Cuza Iaşi Secţ. I a Mat **40** (1994), 55-61.

[32] **Rosca, R.**: *CR-hypersurfaces champ normal covariant décomposable incluses dans une variété pseudo-riemannianne neutre.* C. R. Acad. Sci. Paris **292** (1981), 287-290.

[33] **Rosca, R.**: *Codimension 2, CR-submanifolds with null covariant decomposable vertical distribution of a nautral manifold M.* Rend. Mat. (7) **2** (1983), 787-797.

[34] **Rosca, R.**: *CR-sous-variétés co-isotropes d'une variété parakaehlerienne.* C. R. Acad. Sci. Paris **298** (1984), 149-151.

[35] **Tazawa, Y.**: *Construction of slant immersions.* **II.** Bull. Belg. Math. Soc. Simon Stevin **1** (1994), 569-576.

[36] **Wells, R. O. Jr**. *Differential Analysis on Complex Manifolds.* Springer, N. York, 1980.

[37] **Yang, J.**: *On slant surfaces with constant mean curvature in $C^2$.* J. Geom. **59** (1997), 184-201.

[38] **Yano, K.; Kon, M.**: *Anti-invariant Submanifolds.* M. Dekker Inc., New York, 1976.