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METHODS FOR THE AERODYNAMICS APPROXIMATION IN THE LAPLACE DOMAIN FOR THE AEROSERVOELASTIC INTERACTIONS STUDIES

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Abstract. *This paper presents a comparison of three methods of approximation for the unsteady generalized aerodynamic forces acting on a flexible aircraft. These methods are: the Least Square LS, the Minimum State MS and the Matrix Padé MP approximations. These approximations are used for the aeroservoelastic interactions studies on a fly-by-wire aircraft, where the aerodynamic forces in the reduced frequency k domain (dynamics and loads) have to be approximated in the Laplace s domain (flight control). The aim is, in this case, to study the effects of the control laws on the flexible aircraft structure.*

In addition, the convergence of minimum state approximation method MS of unsteady generalised aerodynamic forces in the equation of motion of flexible aircraft is shown by means of an original feature. At each iteration, an optimal compromise is chosen between the present and the last iteration.

1. INTRODUCTION

The aeroservoelastic interactions concern mainly the interaction between the three main following disciplines: aerodynamics, aeroelasticity and servo-controls. Progress in this multidisciplinary area has demanded optimisation of aerodynamic methods in their capability to generate s-domain aerodynamics from k-domain aerodynamics.

Aeroservoelasticity establishes the s-domain as a base, which can be obtained from the k-domain aerodynamics by means of several rational approximation methods [1, 2, 3, 4], such as:

- The conventional Least Square LS
- The Modified Matrix Padé MP
- The Minimum State MS

The conventional Least Square LS method has been used in the aeroservoelastic computer program called ADAM[18,], where its capability is to determine Padé approximations of any order such that the sum of numerator and denominator terms does not exceed 15.

The state space equations include augmented states that represent the aerodynamic lags; their number is dependent on the number of denominator roots in the rational approximation. The aerodynamic entire matrix has been approximated by a ratio of matrix polynomials [4]. In the Matrix Padé MP approximation method [5], each term of the aerodynamic matrix may be approximated by a polynomial ratio in s. However, it has been found also that common denominator roots in defining the corresponding polynomials are also effective. Other modifications of the MP method were suggested in [6,7].

A higher number of denominator roots is required in the minimum state MS approximation method [8]. In this method, the Number of augmented states is equal to the number of the denominator roots.

In [9,10,11] the capabilities for enforcing or relaxing equality constraints were included in the LS, MP and MS methods. These capabilities were abbreviated ELS, EMP and EMS, and they were introduced in the aeroservoelastic computer program called Interaction of Structures, Aerodynamics and Controls (ISAC) [17]. The minimum state approach (MIST) is selected recently in the ASTROS computer program [12]. This method offers savings in the number of added states with little or no penalty in the accuracy of modelling the aerodynamic forces. However, its applicability to the unsteady aerodynamics in the transonic and hypersonic regimes remains to be established.

Comparisons are given between the existing aerodynamic optimisation methods MP, LS, MS aerodynamic optimisation methods. As the Minimum-State MS approach gives an ill-conditioned system, in this paper an additional new feature is presented to solve this problem. This mechanism consists of choosing, at each iteration, an optimal compromise between the present and the last iteration.

2. EQUATIONS OF MOTION

For this reason, one should approximate the aerodynamic forces $Q(k, M)$ by a rational polynomial $Q(s)$. The equations of motion for a flexible aircraft under the influence of unsteady aerodynamic loads is described by the following matrix equation

$$\mathbf{M}\,\ddot{\mathbf{q}} + \mathbf{C}\,\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} + q_{dyn}\mathbf{A}_e(k)\,q = \mathbf{P}(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{K}$, $\mathbf{C}$ are the generalised mass, elastic stiffness and damping matrices, q_{dyn} is the dynamic pressure, $q_{dyn} = 0,5\rho V^2$, where ρ is the air density and V is the true airspeed, $k = \omega b/V$ is the reduced frequency where ω is the natural frequency, b is the wing semichord length, $\mathbf{A}_e(k)$ is the aerodynamic influence coefficient matrix for a given Mach number M and a set of k_l values, $\mathbf{q}$ is the displacement vector, $\mathbf{P}(t)$ is the external forcing function, s is the Laplace variable $(=j\omega)$.

A solution of the related free-vibration problem

$$\mathbf{M}\,\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = 0 \tag{2}$$

yields the desired roots ω and vectors ϕ. By applying the transformation

$$\mathbf{q} = \Phi\eta \tag{3}$$

to equation (1) and pre-multiplying both sides by Φ^T and Φ, the generalised equation of motion is derived by

$$\hat{\mathbf{M}}\ddot{\eta} + \hat{\mathbf{C}}\dot{\eta} + \hat{\mathbf{K}}\eta + q_{dyn}\mathbf{Q}(\mathbf{k})\eta = \hat{\mathbf{P}}(t) \tag{4}$$

in which $\hat{\mathbf{M}} = \Phi^T \ \mathbf{M} \ \Phi$, and so forth. The modal matrix $\Phi = [\Phi_r \ \Phi_e \ \Phi_\delta]$ and the generalised coordinates $\eta = [\eta_r \ \eta_e \ \eta_\delta]$ incorporate rigid body, elastic, and control surface motions, respectively.

The order of each generalised matrix is equal to the number of retained modes. For the aeroservoelastic analysis, the aerostructural problem is recast in the Laplace domain, by calculating the Laplace transform representation of equation (4) :

$$\left[\hat{\mathbf{M}}s^2 + \hat{\mathbf{C}}s + \hat{\mathbf{K}} + q_{dyn}\mathbf{Q}(\bar{s})\right]\eta(s) = \hat{\mathbf{P}}(s) \tag{5}$$

where $\bar{s} = sb/V$ is the normalised Laplace variable. In this context, the matrix $\mathbf{Q}(\bar{s})$ can be represented by a ratio of polynomials in $\bar{s}$. In this paper, the generalised aerodynamic forces $\mathbf{Q}(\bar{s})$ are curve fitted using Least Squares (LS), Matrix Padé (MP), and Minimum-State (MS) approximations.

Next, we are going to explain the three theories of aerodynamic approximation:

2.1 LEAST SQUARE LS APPROXIMATION

Least-Square Approximation is based on the following model

$$\hat{\mathbf{Q}}(\bar{s}) = \mathbf{A}_0 + \mathbf{A}_1\bar{s} + \mathbf{A}_2\bar{s}^2 + \sum_n \mathbf{A}_{(n+2)}\frac{\bar{s}}{\bar{s} + b_n} \tag{6}$$

which can be rewritten in the frequency domain as follow

$$\hat{\mathbf{Q}}(\vec{j}k) = \mathbf{A}_0 + \mathbf{A}_1\vec{j}k - \mathbf{A}_2k^2 + \sum_n \mathbf{A}_{(n+2)}\frac{\vec{j}k}{\vec{j}k + b_n} \tag{7}$$

where $\hat{\mathbf{Q}}$ is approximation of matrix $\mathbf{Q}$, k is reduced frequency variable, $\mathbf{A}_i$ are the coefficients matrices and b_n are lag coefficients of the approximation model. With this model, the approximation can be found by minimising the following objective function

$$J = \sum_i \sum_j \sum_l W_{ijl}^2 |Q_{ij}(\vec{j}k_l) - Q_{ij}(\vec{j}k_l)|^2 \tag{8}$$

where k_l is the lth normalised frequency and $\mathbf{W}$ is the weighting table which is usually chosen as

$$W_{ijl} = \frac{1}{\max(1, |Q_{ij}(\vec{j}k_l)|)} \tag{9}$$

If lag coefficients b_n are fixed, the objective function with weighting table given by eq. (9) is linear quadratic so that it can be easily minimised. The solution of this minimisation problem is given by the following equation

$$\begin{bmatrix} A_{0ij} \\ A_{1ij} \\ \vdots \end{bmatrix} = \left\{ \sum_l W_{ijl}^2 \left(\mathbf{B}_{Rl}^T \mathbf{B}_{Rl} + \mathbf{B}_{Il}^T \mathbf{B}_{Il} \right) \right\}^{-1} \left\{ \sum_l W_{ijl} \left(\mathbf{B}_{Rl}^T \mathbf{Q}_{Rijl} + \mathbf{B}_{Il}^T \mathbf{Q}_{Iijl} \right) \right\} \tag{10}$$

where $Q_{Rijl} = \mathrm{Re}\left\{Q_{ij}(\bar{j}k_l)\right\}$, $Q_{Iijl} = \mathrm{Im}\left\{Q_{ij}(\bar{j}k_l)\right\}$,

$$\mathbf{B}_{Rl} = \left[-1 \; 0 \; k_l^2 \quad \frac{-k_l^2}{k_l^2 + b_1^2} \quad \frac{-k_l^2}{k_l^2 + b_2^2} \; \cdots \right] \tag{11}$$

and

$$\mathbf{B}_{Il} = \left[0 \; -k_l \; 0 \quad \frac{-k_l b_1}{k_l^2 + b_1^2} \quad \frac{-k_l b_2}{k_l^2 + b_2^2} \; \cdots \right] \tag{12}$$

This solution depends of fixed lag coefficients bi so that it can be replaced in objective function (8) to form a new objective function according to bn variables. This non-linear objective function can then be minimised using non-linear programming techniques. In this paper, one has chosen to use the Quasi-Newton method from the Matlab Optimisation Toolbox.

2.2 MATRIX PADÉ MP APPROXIMATION

Padé is similar to the Least-Square Approximation but there are different b_n for each column so that it use the following Model [11] :

$$\hat{\mathbf{Q}}_j(\bar{s}) = \mathbf{A}_{0j} + \mathbf{A}_{1j}\bar{s} + \mathbf{A}_{2j}\bar{s}^2 + \sum_n \mathbf{A}_{(n+2)j} \frac{\bar{s}}{\bar{s} + b_{nj}} \tag{13}$$

which can be rewritten in the frequency domain as follows

$$\hat{\mathbf{Q}}_j(\bar{j}k) = \mathbf{A}_{0j} + \mathbf{A}_{1j}\bar{j}k - \mathbf{A}_{2j}k^2 + \sum_n \mathbf{A}_{(n+2)j} \frac{\bar{j}k}{\bar{j}k + b_{nj}} \tag{14}$$

where $\hat{\mathbf{Q}}_j$ is approximation of the jth column of matrix $\mathbf{Q}$, $\mathbf{A}_{ij}$ are the coefficients matrices of jth column of the matrix $\mathbf{A}_i$ and b_{nj} are lag coefficients of the jth column of approximation model. With this model, the approximation can be found by minimising the following objective function:

$$J_j = \sum_i \sum_l W_{ijl}^2 |Q_{ij}(jk_l) - \hat{Q}_{ij}(jk_l)|^2 \tag{15}$$

where k_l is the lth normalised frequency and $\mathbf{W}$ is the weighting table which is usually chosen as Eq (9). The minimisation of the objective function (15) is very similar to minimisation of Eq (8). The only difference between the solution of Least-Square method and Padé method is matrices $\mathbf{B}_{Rl}$ and $\mathbf{B}_{Il}$ which are different for each column

2.3 MINIMUM STATE MS APPROXIMATION

Minimum State MS approximation is relatively different of Matrix Padé MP and Least Square LS approximations. This approximation is based on the following model:

$$\hat{\mathbf{Q}}\,(\bar{s}) = \mathbf{A}_0 + \mathbf{A}_1\bar{s} + \mathbf{A}_2\bar{s}^2 + \mathbf{D}[\bar{s}\mathbf{I} - \mathbf{R}]^{-1}\mathbf{E}\bar{s} \tag{16}$$

which can be rewritten in the frequency domain as follows

$$\hat{\mathbf{Q}}(\vec{j}k) = \mathbf{A}_0 + \mathbf{A}_1\vec{j}k - \mathbf{A}_2k^2 + \mathbf{D}[\bar{s}\mathbf{I} - \mathbf{R}]^{-1}\mathbf{E}\bar{s} \tag{17}$$

where $\mathbf{R}$ is diagonal matrix of lags. As for the Least Square LS and Matrix Padé MP approximation, the objective is to minimise the cost function (8). Usually, the following constraints are applied before to minimise objective function [13],

$$\mathrm{Re}\left\{\hat{\mathbf{Q}}(0)\right\} = \mathrm{Re}\left\{\mathbf{Q}(0)\right\}$$

$$\mathrm{Re}\left\{\hat{\mathbf{Q}}(\vec{j}k_f)\right\} = \mathrm{Re}\left\{\mathbf{Q}(\vec{j}k_f)\right\}$$

$$\mathrm{Im}\left\{\hat{\mathbf{Q}}(\vec{j}k_g)\right\} = \mathrm{Im}\left\{\mathbf{Q}(\vec{j}k_g)\right\}$$

where $k_f \neq 0$ and $k_g \neq 0$. By using the following equality

$$\left(\vec{j}k\mathbf{I} - \mathbf{R}\right)^{-1} = \left(k^2\mathbf{I} + \mathbf{M}^2\right)^{-1}\left(-\vec{j}k\mathbf{I} - \mathbf{R}\right) \tag{18}$$

These constraints transform the model as follow

$$\hat{\mathbf{Q}}(\vec{j}k_l) = \mathbf{B}_{Rl} + \mathbf{D}\mathbf{C}_{Rl}\mathbf{E} + \vec{j}\,(\mathbf{B}_{Il} + \mathbf{D}\mathbf{C}_{Il}\mathbf{E}) \tag{19}$$

where

$$\mathbf{B}_{Rl} = \mathbf{Q}_R(0) - \tfrac{k_l^2}{k_f^2}\left[\mathbf{Q}_R(0) - \mathbf{Q}_R(k_f)\right]$$

$$\mathbf{C}_{Rl} = \mathbf{D}\left[\left(k_l^2\mathbf{I} + \mathbf{R}^2\right)^{-1} - \left(k_f^2\mathbf{I} + \mathbf{R}^2\right)^{-1}\right]k_l^2$$

$$\mathbf{B}_{Il} = \tfrac{k_l}{k_g}\mathbf{Q}_I(k_g)$$

$$\mathbf{C}_{Il} = \mathbf{D}\left[\left(k_G^2\mathbf{I} + \mathbf{R}^2\right)^{-1} - \left(k_L^2\mathbf{I} + \mathbf{R}^2\right)^{-1}\right]\mathbf{R}k_l$$

with

$\mathbf{Q}_R(k) = \mathrm{Re}\left\{\mathbf{Q}(\vec{j}k)\right\}$ and $\mathbf{Q}_I(k) = \mathrm{Im}\left\{\mathbf{Q}(\vec{j}k)\right\}$.

Then the objective function can be minimised. However, the objective is now non linear according to $\mathbf{E}$ and $\mathbf{D}$ matrices. The solution can then be obtained using iterative linear quadratic solution. In fact $\mathbf{D}$ matrix is fixed and the linear quadratic problem is solved according to $\mathbf{E}$ matrix. Next, $\mathbf{E}$ matrix is fixed and the linear quadratic problem is solved according to $\mathbf{D}$ matrix. This scheme is repeated until the convergence is attained.

To solve the linear quadratic problem according to $\mathbf{E}$ matrix with $\mathbf{D}$ matrix fixed. The objective function is rewritten by column as follows

$$J = \sum_j J_j \tag{20}$$

where column error is

$$J_j = \sum_l \mathbf{e}_l^*(k_l)\mathbf{W}_{jl}^2\mathbf{e}_j(k_l) \tag{21}$$

with

$$\mathbf{e}_j(k_l) = \mathbf{Q}_j(\bar{j}k_l) - \hat{\mathbf{Q}}_j(\bar{j}k_l) \tag{22}$$

The objective function (20) can then be minimised by minimising objective function (21) for each column. According to eq. (19), solve minimisation problem (21) is equivalent to solve the following equation for each column of $\mathbf{E}$ [14]

$$\mathbf{A}e\mathbf{E}_j = \mathbf{B}e \tag{23}$$

where

$$\mathbf{A}e = \sum_l \mathbf{C}_{Rl}^T\mathbf{D}^T\mathbf{W}_{jl}^2\mathbf{D}\mathbf{C}_{Rl} + \mathbf{C}_{Il}^T\mathbf{D}^T\mathbf{W}_{jl}^2\mathbf{D}\mathbf{C}_{Il}$$

$$\mathbf{B}e = \sum_l \mathbf{C}_{Rl}^T\mathbf{D}^T\mathbf{W}_{jl}^2\left(\mathbf{Q}_{Rj}(k_l) - \mathbf{B}_{Rjl}\right) + \mathbf{C}_{Il}^T\mathbf{D}^T\mathbf{W}_{jl}^2\left(\mathbf{Q}_{Ij}(k_l) - \mathbf{B}_{Ijl}\right)$$

Similarly, the error can be rewritten as row error and the linear quadratic problem according to $\mathbf{D}$ Matrix is equivalent to solve the following equation for each row of $\mathbf{D}$:

$$\mathbf{D}_i\mathbf{A}d = \mathbf{B}d \tag{24}$$

where

$$\mathbf{A}d = \sum_l \mathbf{C}_{Rl}\mathbf{E}\mathbf{W}_{jl}^2\mathbf{E}^T\mathbf{C}_{Rl}^T + \mathbf{C}_{Il}^T\mathbf{E}\mathbf{W}_{jl}^2\mathbf{E}^T\mathbf{C}_{Il}^T$$

$$\mathbf{B}d = \sum_l \left(\mathbf{Q}_{Rj}(k_l) - \mathbf{B}_{Rjl}\right)\mathbf{W}_{jl}^2\mathbf{E}^T\mathbf{C}_{Rl}^T + \left(\mathbf{Q}_{Ij}(k_l) - \mathbf{B}_{Ijl}\right)\mathbf{W}_{jl}^2\mathbf{E}^T\mathbf{C}_{Il}^T$$

The MS approximation is generally better than Least Square and Padé approximations. However, when the number of lags is greater than the number of frequencies, the problem is usually ill conditioned. To solve this problem, we consider two additional features: 1) At each iteration, the linear quadratic problems according to $\mathbf{E}$ matrix and after that to $\mathbf{D}$ matrix are solved using singular value decomposition [15]. 2) At each iteration, the optimal compromise between the present and the last iteration is chosen to ensure the convergence of algorithm.

2.3.1 SINGULAR VALUE DECOMPOSITION SVD

Equations (23) and (24) have the following form

$$\mathbf{A}\mathbf{x} = \mathbf{B} \tag{25}$$

where $\mathbf{x}$ is unknown vector. When this system is ill conditioned, the least square solution can be obtained using singular value decomposition of $\mathbf{A}$. In fact, $\mathbf{A}$ can be decomposed as follows:

$$\mathbf{A} = \mathbf{U}\mathbf{S}\mathbf{V}^T$$

where $\mathbf{U}$ and $\mathbf{V}$ are orthogonal matrices and $\mathbf{S}$ is diagonal. The diagonal of $\mathbf{S}$ matrix contains, in order, the singular value of $\mathbf{A}$ so that, if rank($\mathbf{A}$) = r < n, the n-r last elements of diagonal are null. Using this decomposition, the least square solution of eq. (25) can be obtained as follows:

$$x = \sum_{i=1}^{r} \frac{\mathbf{u}_i^T \mathbf{B} \mathbf{v}_i}{s_i} \quad (26)$$

where $\mathbf{u}_i$ and $\mathbf{v}_i$ are respectively the column of matrix $\mathbf{U}$ and $\mathbf{V}$ and s_i is the i'Th element of diagonal matrix $\mathbf{S}$. This least square solution can be obtained easily with the standard function of Matlab software.

2.3.2 OPTIMAL COMPROMISE

Let $\mathbf{E}_{(p)}$ and $\mathbf{E}_{(p-1)}$ be the optimal solution at present and last iteration so that the optimal compromise between present and last iteration is obtained by minimising the objective function (21) according to a with

$$\mathbf{E} = \alpha \mathbf{E}_{(p-1)} + (1-\alpha)\mathbf{E}_{(p)} \quad (27)$$

Similarly, for $\mathbf{D}$ matrix, the objective function can be minimised according to a with

$$\mathbf{D} = \alpha \mathbf{D}_{(p-1)} + (1-\alpha)\mathbf{D}_{(p)} \quad (28)$$

For these scalar minimisation, we have used the scalar minimisation algorithm included in the Matlab Optimisation toolbox.

3. APPLICATION

The finite element model of the symmetric half of the aircraft [16] is used for the vibration analysis. Only the antisymmetric case is here presented.

Forty vibration modes were considered as follows: the first three modes are the three perfect rigid body modes Φ_{PR} (Y-translation, X-rotation roll, and Z-rotation yaw about the centre of gravity CG), thirty-five flexible bending and torsion modes Φ_F, and two rigid control modes Φ_C (aileron and rudder deflections).

The Doublet Lattice method which is an unsteady aerodynamic lifting surface method was used to obtain the values for the generalised aerodynamic forces $\mathbf{Q}(k, M)$ as functions of reduced frequency and Mach number M. To calculate the pressure distribution on oscillating lifting surfaces (wing, horizontal and vertical tail) undergoing simple harmonic motions, the lifting surfaces are subdivided into arrays of trapezoidal boxes arranged in strips parallel to the airstream. The lifting surfaces are further represented by lattices of doublets located at the quarter-chord of each box. The downwash boundary condition is satisfied at the three-quarter-chord of each box. The downwash is computed from the slope and deflection of each structural mode.

For this purpose, STARS (STructural Analysis RoutineS) program package is used to calculate the modes of vibration and the aerodynamic oscillatory forces $Q(k, M)$ for the following reduced frequencies $k = 0.001, 0.01, 0.02, 0.1, 0.2, 1$ and $M = 0.9$.

The Matlab software is then used for computation of optimisation algorithm. LS, MP and MS are considered for approximation of Q. Figure 1 shows total error for LS, MP and

MS model of the same order (160). These results shows us clearly the superiority of the MS model.

Figures 2 and 3 show the comparison between the unsteady generalised aerodynamic forces approximation $Q(s)$ and the true unsteady generalised aerodynamic forces $Q(k, M)$. Figure 2 shows the unsteady aerodynamic forces for the vertical fin bending mode FINB. Figure 3 shows the unsteady aerodynamic forces for the wing second bending mode W2B. Figure 4 shows the convergence error versus the number of iterations.

4. CONCLUSIONS

A comparison of these methods shows the superiority of the MS approximations. In fact, Figure 1 shows that the errors for those 3 approximations are about the same but the MS approximation needed only 20 state variables whereas MP and LS approximations needed 120 state variables. However, there is a problem with the Minimum-State MS approach: the system is ill-conditioned when the number of approximation lags is larger than the number of reduced frequencies. Then, this paper presents an additional new feature to solve this problem. This mechanism consists of choosing, at each iteration, an optimal compromise between the present and the last iteration.

Figure 1: Approximation errors for the three methods LS, MP and MS

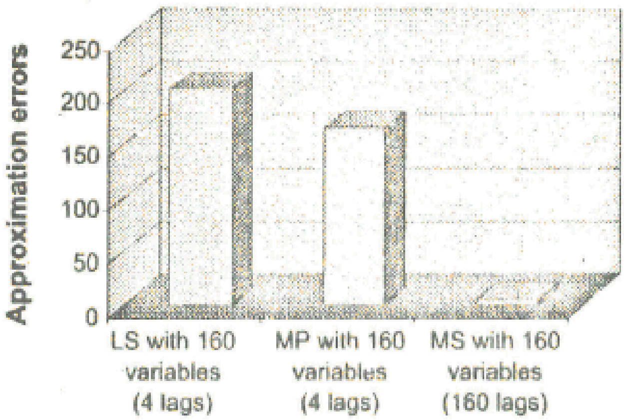


Figure 2: Aerodynamics forces representation for the vertical fin bending FINB

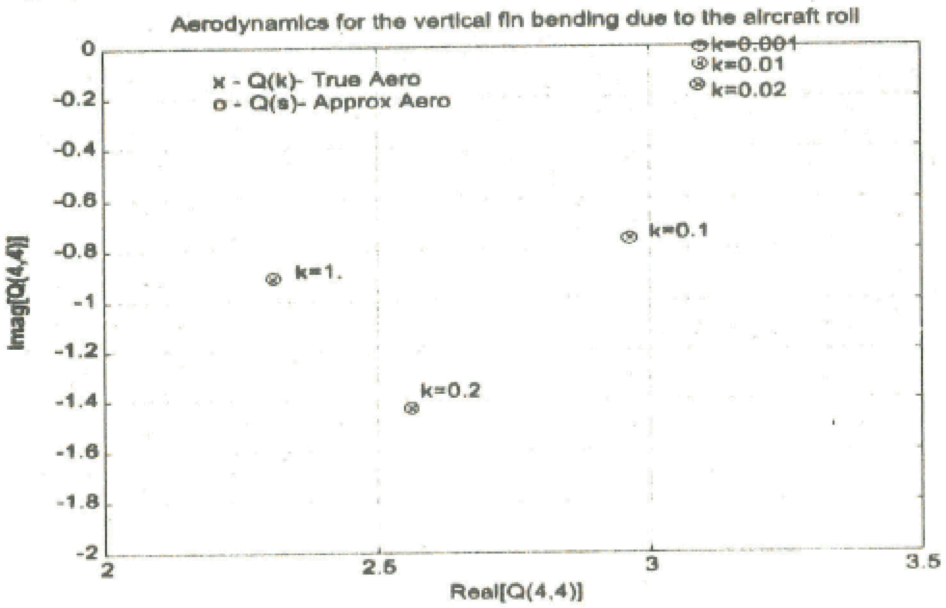


Figure 3: Aerodynamics forces representation for the wing second bending W2B

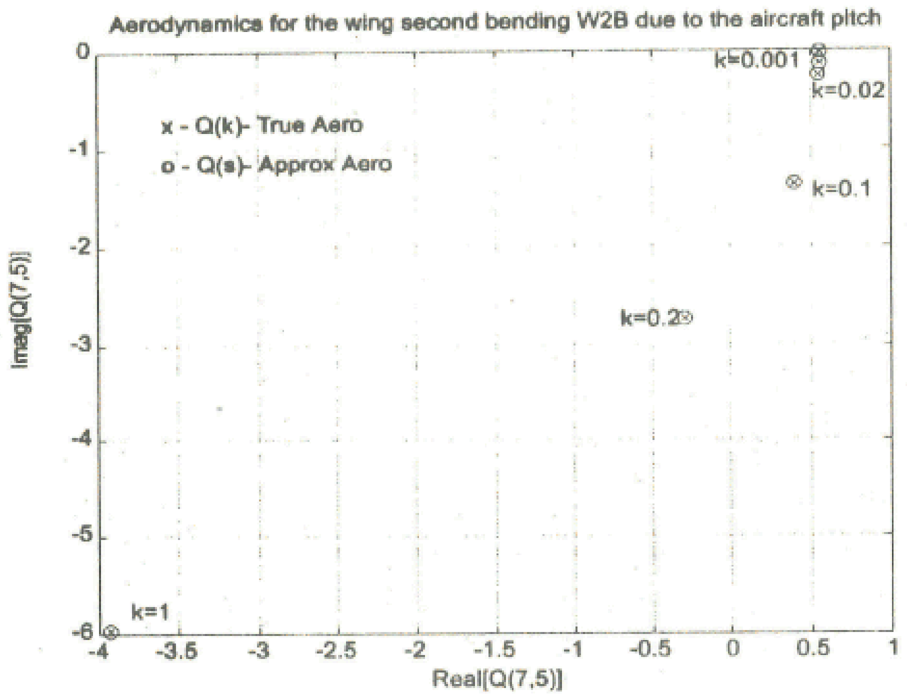
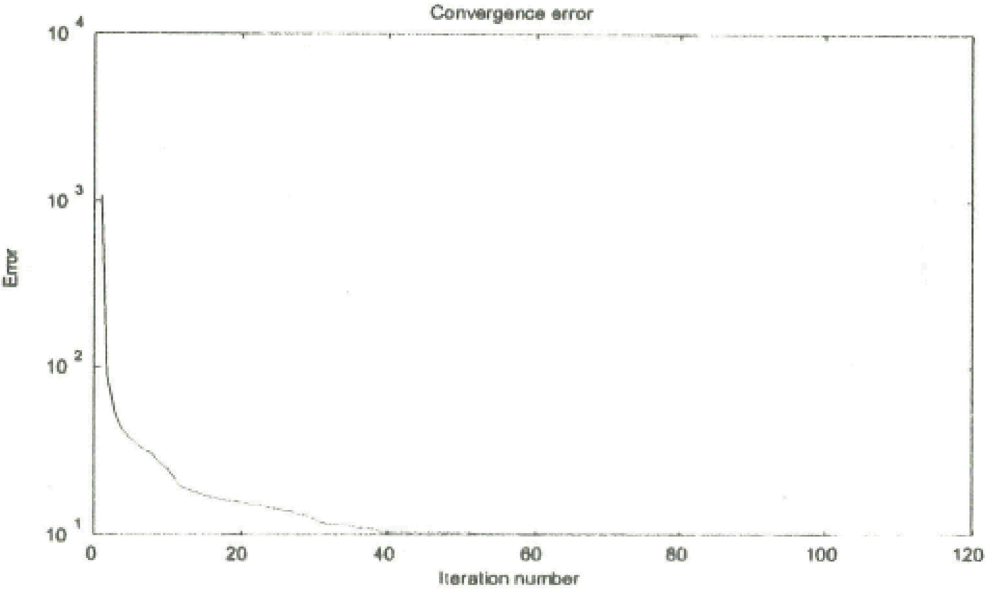


Figure 4: Convergence error versus the iteration number



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