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CHANNELED SPECTRA OF ANISOTROPIC LAYERS. BIREFRINGENCE DISPERSION DETERMINATION

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Abstract. The conditions of channeled spectra appearance were theoretically established. The values of the birefrigerence and of the birefrigerence dispersion in the visible range were determined for a solution of poly (phenyl methacrylic) esther of cetyloxybenzoic acid in tetrachlormethane with induced anisotropy by an external electric field.

Keywords: channeled spectra, PPMAECOBA solution in TCM, birefrigerence dispersion.

1 Introduction

An anisotropic substance has only three values of the refractive indices if it is related to the principal system of coordinates $(aobc)$ [1–4]. Two values of the refractive index characterize an uniax anisotropic layer in the principal system of coordinates; $n_c = n_e$ for linear polarized waves with electric field intensity parallel to the optical axis oc and $n_a = n_o$ for linear polarized waves with electric field intensity parallel to the principal plane aoc. In this case the difference $\Delta n = n_c - n_e$ determines the birefringence of the substance [4, p. 52].

The refractive index generally varies with light wave number. It results that the birefringence must depend on the light spectral composition. The phenomenon is called dispersion of the birefringence.

One can determine the variation of the birefringence with the light wave numbers from the channeled spectra that consist from a succession of maximum and minimum intensities of energy in the wave number scale [1, p. 129].

We intend to establish the conditions of the channeled spectra appearance and to calculate the dispersion from an experimentally obtained channeled spectrum for an uniax anisotropic layer when light propagates along a principal axis, differing from oc axis, that is generally considered as being parallel with the optical axis of the anisotropic medium.

2 THEORETICAL ASPECTS

Let's consider a system composed from two identical polarizers P_1 and P_2 and an anisotropic layer L disposed between them (Fig. 1). A natural monochromatic light passing through this system changes its polarization state. After polarizer P_1, light becomes plan polarized

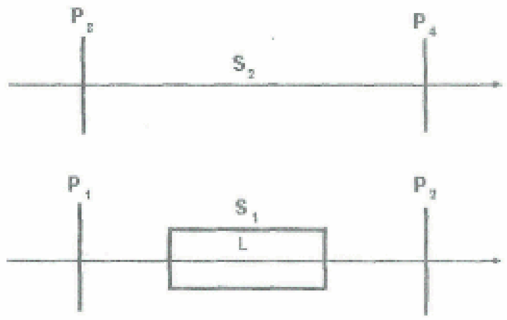

Fig. 1. The device attached to a double beam spectrophotometer for obtaining channeled spectra. $P_1 - P_4$ are identical polarizers ($P_1 \perp P_2$ and $P_3 || P_4$), L is the anisotropic layer, S_1 is the measure beam and S_2 is the reference one.

with the electric field intensity parallel to the transmission direction of P_1. Let $\vec{u}_{P_1}$ be the versor characterizing the polariser transmission direction.

Let $aobc$ be the principal system of axes in the anisotropic layer and the light propagation direction orientated along the ob axis. In these conditions the electromagnetic wave that propagates in the anisotropic layer has components only in the plane perpendicular on the propagation direction.

Let's consider the electric field intensity after P_1 as being:

$$\vec{e}_{P_1} = E_{P_1} \cdot \vec{u}_{P_1} \cdot e^{\mathrm{i}(\omega t + \varphi_0)} \tag{2.1}$$

where E_{P_1} is the amplitude of the electric field intensity, ω – the angular speed of the electromagnetic wave and φ_0 is the initial phase.

The flux density of light after P_1 is a half from the flux density in the incident natural light [3, p. 22]:

$$\phi_{P_1} = \frac{1}{c\mu_0} \vec{e}_{P_1} \cdot \vec{e}^{\,*}_{P_1} = \frac{1}{2} \phi_i \tag{2.2}$$

Relation (2.2) shows that the transmission factor of the polarizer P_1 for the natural light is 0.5.

One may consider that the linear polarized light, that propagates along axis ob of the anisotropic layer, is decomposed in two linear polarized waves with the intensities of the electric field $\vec{e}_a$ and $\vec{e}_c$ perpendicular between themselves (see Fig. 2) and parallelly orientated to the principal axes oa and oc of the anisotropic layer.

Let $\vec{a}_0$ and $\vec{c}_0$ be the versors of the main directions oa and oc. The analytical expressions of these components, at entrance in the anisotropic layer, are:

$$\begin{aligned} \vec{e}_a &= (\vec{e}_{P_1} \cdot \vec{a}_0) \cdot \vec{a}_0 \\ \vec{e}_c &= (\vec{e}_{P_1} \cdot \vec{c}_0) \cdot \vec{c}_0 \end{aligned} \tag{2.3}$$

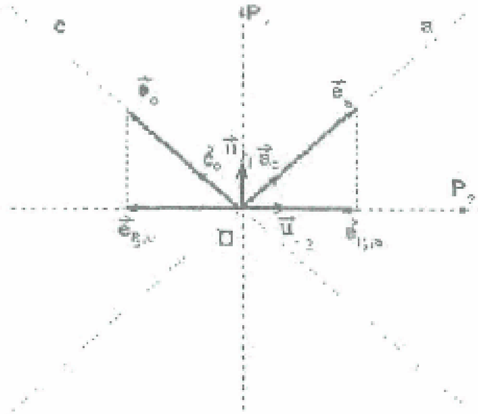

Fig. 2. The transmission directions of the polarizers P and A and the principal axes oa and oc of the anisotropic layer

If the anisotropic substance is transparent for visible radiation, the emergent components from the anisotropic layer have the expressions:

$$\vec{e}_a(L) = E_{P_1} \cdot (\vec{u}_{P_1} \cdot \vec{a}_0) \cdot \vec{a}_0 \cdot e^{i(\omega t - k_a L + \varphi_0)}$$
$$\vec{e}_c(L) = E_{P_1} \cdot (\vec{u}_{P_1} \cdot \vec{c}_0) \cdot \vec{c}_0 \cdot e^{i(\omega t - k_c L + \varphi_0)} \tag{2.4}$$

In relation (2.4), $k_a = \frac{2\pi}{\lambda_0} n_a$; $k_c = \frac{2\pi}{\lambda_0} n_c$ are the module of the propagation vector [4, p. 53] for the plane polarized waves having the electric field intensities orientated paralelly with principal axes oa and oc, L is the thickness and $\Delta n = n_c - n_a$ is the birefringence of the anisotropic layer.

Let $\vec{u}_{P_2}$ be the versor of the transmission direction of the polariser P_2.The emergent electric field from the polarizer P_2 has only components parallelly orientated to its transmission direction. After P2 , the two components that can leave the system are:

$$\vec{e}_{P_2,a} = \vec{e}_a(L) \cdot \vec{u}_A \cdot \vec{u}_A = E_{P_1}(\vec{u}_{P_1} \cdot \vec{a}_0)(\vec{a}_0 \cdot \vec{u}_{P_2}) \cdot \vec{u}_{P_2} \cdot e^{i(\omega t - k_a L + \varphi_0)} \tag{2.5}$$
$$\vec{e}_{P_2,c} = \vec{e}_c(L) \cdot \vec{u}_A \cdot \vec{u}_A = E_{P_1}(\vec{u}_{P_1} \cdot \vec{c}_0)(\vec{c}_0 \cdot \vec{u}_{P_2}) \cdot \vec{u}_{P_2} \cdot e^{i(\omega t - k_c L + \varphi_0)}$$

The angles between optical axis oc of the anisotropic layer and the transmission directions of the polarisers are $\sphericalangle(\vec{u}_{P_1}, \vec{c}_0) = \alpha$ and $\sphericalangle(\vec{u}_{P2}, \vec{c}_0) = \beta$. Expressing the scalar products of the corresponding vectors, relation (2.5) becomes:

$$\vec{e}_{P_2,a} = \vec{e}_a(L) \cdot \vec{u}_{P_2} \cdot \vec{u}_{P_2} = E_{P_1} \cdot \cos\alpha \cdot \cos\beta \cdot \vec{u}_{P_2} e^{i(\omega t - k_a L + \varphi_0)} \tag{2.6}$$
$$\vec{e}_{P_2,c} = \vec{e}_c(L) \cdot \vec{u}_{P_2} \cdot \vec{u}_{P_2} = E_{P_1} \cdot \sin\alpha \cdot \sin\beta \cdot \vec{u}_{P_2} e^{i(\omega t - k_c L + \varphi_0)}$$

The electric field intensity of the emergent light from the system results as a vectorial sum of the components from (2.6):

$$\vec{e}_{P_2} = \vec{e}_{P_2,a} + \vec{e}_{P_2,c} \tag{2.7}$$