

## On Some Remarkable Results of R.Miron

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**Abstract.** The decisive role of non-linear connection, introduced by R. Miron on the total space  $E$  (or  $E = TM$ ) of a vector bundle  $\xi = (E, \pi, M)$  is now largely recognized. The use of a non-linear connection is of help in keeping the geometrical meaning of various concept on  $E$ , on the Finsler spaces, on the Lagrange space and on the Hamilton spaces. These spaces are more and more important for applications in theoretical physics and the gravitational theories. In the following we will present some of our results obtained between 1984-2002, that were inspired by the papers and the books of R. Miron.

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### 1 Some $d$ -connections on the total space $E$

In the following the fundamental notions and notations from the book of R. Miron ([1]) are used together with author's notations from [10]. Let  $\xi = (E, \pi, M)$  be a vector bundle where the base manifold is an  $n$ -dimensional paracompact  $C^\infty$ -differentiable manifold  $M_n = [M, [A], \mathbb{R}^n]$ , whose type fibre is  $F = \mathbb{R}^n$ , endowed with a non-linear connection  $N$ . This is given by a horizontal distribution  $H : u \rightarrow H_u E$  which is supplementary to the vertical distribution  $V : u \rightarrow V_u E$ , which induces a unique decomposition  $X = hX + vY, \forall X \in \mathcal{X}(E)$  and  $\omega = h\omega + v\omega, \forall \omega \in \wedge_1(E)$ . The concept of  $d$ -connection on  $E$  is due to R. Miron.

**Definition 1.** A  $d$ -connection on  $E$  is a linear connection  $D$  which preserves by parallelism the horizontal and the vertical distributions on  $E$ .

Using the differential manifold structure  $E_{n+m}$  and the above concept of the  $d$ -connection  $D$  on  $E$ , one can study the geometry of the space  $(E_{n+m}, D)$ . We will mention only some results which extend the geometry of the space  $(M_n, \nabla)$  with the linear connection  $\nabla$  to the geometry of the space  $(E_{m+n}, D)$ .

**Definition 2.** ([11]) A system of tensor equations

$$h\Omega(X, vY) = \rho_2(Y)hX, \quad v\Omega(X, hY) = \rho_1(Y)hX, \quad \forall X, Y \in \mathcal{X}(E) \quad (1)$$

with  $\Omega$  as unknown,  $\Omega \in \mathcal{T}(\frac{1}{2})(E)$  and  $\rho_1 = h\rho, \rho_2 = v\rho, \rho$  an arbitrary given covector on

$E$ , is called the  $\Omega(\rho)$ -system.

The system

$$h\tau(X, vY) = 0, \quad v\tau(X, hY) = 0, \quad \forall X, Y \in \mathcal{X}(E) \quad (2)$$

is called the homogeneous  $\tau$ -system.

The general solution  $\tau$  of a  $\tau$ -system and for  $\Omega(\rho)$ -system are obtained. Some special classes of solutions  $\Omega$  of an  $\Omega_0$  are studied. Using these solutions are studied the special transformations  $D \rightarrow \overline{D}$ , of the  $d$ -connections on  $E$ . The general projective connection transformations and the special projective transformations are obtained. The invariants of Schouten, of Weyl and of Thomas types on  $E$  are generalized. As application the problem of the prolongation of the projective geometry of the paths to  $(E_{n+m}, D)$  is solved. The parallelism of the direction along every curve  $\gamma$  in the space  $(E_{n+m}, D)$  are studied using the special solution of the equations (1). The methods given by R. Miron are used. For the particular paths  $\gamma$  that are autoparallel of  $D$ , it is possible to define an affine parameter  $s$  of  $\gamma$  and a projective parameter using the generalized coefficients  $J_{\beta\gamma}^{\alpha}$  given in [10].

If  $E = TM$  the new invariants of the projective types are obtained. Using the  $\tau$ -system the following result is obtained :

**Theorem 1.** *If  $(E_{n+m}, D)$  and  $(E_{n+m}, \overline{D})$  are two spaces with the  $d$ -symmetric connections  $D$  and  $\overline{D}$  then a transformations  $\varphi : E_{n+m} \rightarrow E_{n+m}$  is an affine transformation if and only if the paths are the same for the connections  $D$  and  $\overline{D}$ .*

We know that for the space with linear connection  $D$  the autoparallel curves are given by the symmetric part (the mean connection  $D_0$ ) of  $D$ . But, the symmetric part of a  $d$ -connection  $D$  is generally not a  $d$ -connection. A necessary conditions is that the horizontal distribution  $H$  being integrable. It follows that for the study of the affine transformation  $\varphi : E_{n+m} \rightarrow E_{n+m}$  a special attention is necessary if  $\varphi$  is a infinitesimal coliniation. The property from the Theorem 1 suggests the projective character of the symmetric connections. For  $D \neq \overline{D}$  we set the following:

**Definition 3.** *If  $D$  and  $\overline{D}$  are two connections on  $E$  then the transformation  $\tau : D \rightarrow \overline{D}$  given by*

$$\overline{D}_X Y = D_X Y + \rho_1(X)hX + \rho_1(Y) + \rho_2(X)vX + \rho_2(Y)vX, \quad \forall X, Y \in \mathcal{X}(E) \quad (3)$$

is called the  $(hh - vv)$ -transformation.

**Theorem 2.** *A  $(hh - vv)$ -transformation preserves the class of  $d$ -connections on  $E$ . If  $D$  is a  $d$ -symmetric connections then  $\overline{D}$  is a  $d$ -symmetric connection.*

Using (3) for a vector bundle  $\xi = (E, \pi, M)$  with  $m = 1$  (the dimension of fibre type of  $\xi$  is one) and some formulas given by R. Miron and M. Anastasiei [1] with the condition  $N(x, y)$  to be positive homogeneous in  $y > 0$ , we have the possibility of projective study. In

this case the local coordinates on  $E$  are  $\{x^i, y^i = x^0\}$ , ( $i = \overline{1, n}$ ) and the transformations  $\{x^i, x^0\} \rightarrow \{x^{i'}, x^{0'}\}$  of the local coordinates on  $E$  are the form ([1])

$$x^{i'} = x^{i'}(x^1, \dots, x^n), \quad \Delta \neq 0; \quad x^{0'} = f(x^1, \dots, x^n)x^0, \quad f \neq 0,$$

where  $\Delta = \det \left( \frac{\partial x^{i'}}{\partial x^i} \right)$ . R. Miron introduced the adapted basis of  $N$ , by  $\{\delta/\delta x^i, \partial/\partial y^a\}$ , ( $i = \overline{1, n}$ ,  $a = \overline{1, m}$ ), (in this case  $m = 1$ ). Using this local basis, we define

$$J_{jkl}^i = \frac{\delta J_{jk}^i}{\delta x^l} - \frac{\delta J_{jl}^i}{\delta x^k} + J_{jk}^r J_{rl}^i - J_{jl}^r J_{rk}^i$$

where  $\{J_{jl}^i\}$  are the generalized projective coefficients given by

$$J_{jk}^i(x, y) = \Gamma_{jk}^i - \frac{1}{n+1} \left\{ \delta_j^i \left( \frac{1}{n-1} T_k + \Gamma_k \right) - \delta_k^i \left( \frac{n}{n-1} T_j - \Gamma_j \right) \right\}$$

where  $T_k = T_{ik}^i, \Gamma_k = \Gamma_{ik}^i$  ( $T$  is the torsion of  $D$  and  $\{\Gamma_{ik}^i, \Gamma_{bk}^a, \Gamma_{jc}^i, \Gamma_{bc}^a\}$  are the local coefficients of the  $d$ -connection  $D$ , in the local adapted basis. If  $D$  is  $d$ -symmetric connection then  $T = 0$  and  $J_{jk}^i$  are analogous with the Thomas projective coefficients  $\pi_{jk}^i$  of a symmetric connection on the basis  $M$ . If  $m = 1$  we have  $\{\Gamma_{ik}^i, \Gamma_{0k}^0, \Gamma_{j0}^i, \Gamma_{00}^0\}$  for a  $d$ -connection. For a projective theory we will choice  $f = \Delta^{-1/n+1}$  for obtaining the geometric character of the notions which will be introduced. One obtained the notion of the normal projective connection  $D$  ([13], [14]). The projective parameter is introduced by the author (see [13], [14]). One obtained the projective theory that prolongs the projective theory of the base  $M$  to total space of the vector bundle.

## 2 Interpretation

Let  $\nabla$  be a linear connection on  $M_n$ ,  $\{\Gamma_{jk}^i\}$  its local coefficients in a coordinate neighborhood  $U$  ( $U, \varphi \in A$  and the tangent bundle  $E = (TM, \pi, M)$ , where  $u \in E$ ,  $\pi(u) = p$ ,  $p \in U$ ,  $u \in \pi^{-1}(U)$  with local coordinates  $\{x^i, y^i\}$ , ( $i = \overline{1, n}$ ). We consider the set of the functions  $N_j^i(x, y) = y^r \Gamma_{rk}^i(x)$ . The vector fields  $\{\delta/\delta x^k = \partial/\partial x^k - N_k^r \partial/\partial y^r\}$  generate the local fibre  $H_u \subset T_u E$  of the horizontal distribution  $H$  which is complementary of the vertical distribution  $V$ . Since  $\nabla$  is a linear connection, the distribution  $H$  is invariant to action of the multiplicative group  $R^*$  of real numbers. Conversely, the non-linear connection  $N$  induces a linear connection  $\nabla$  on  $M$  only in the particular case above considered relative to  $H$ . Using  $\nabla$  and the classic treatment relative to the parallel transport of an vector field along of a curve  $\gamma$ , we obtain an isomorphism  $\varphi_{p_2 p_1} : T_{p_1} \rightarrow T_{p_2}$  with the following properties: If  $C$  is a differentiable curve on  $M$  and  $p_1, p_2, p_3 \in \gamma$  then  $\varphi$  satisfy the conditions:

a)  $\varphi_{p_2 p_1 p_3} = \varphi_{p_2 p_3} \circ \varphi_{p_3 p_1}, \quad \forall p_1, p_2, p_3 \in C.$

b) The isomorphism  $\varphi_{p_1 p_2}$  is the inverse of the isomorphism  $\varphi_{p_2 p_1}$ .

c) The isomorphism  $\varphi$  depends only on the point  $p_1, p_2$  and the curve  $C$ .

This construction is possible for any two points using a piecewise smooth curve, in a similar way. It follows that a general isomorphism  $\varphi$  with the properties a), b), c) it is more general than the isomorphism defined by  $\nabla$ . Let  $\gamma : I \rightarrow M$  be a differentiable curve and  $M_1 = [\gamma, [A_\gamma], R]$  and  $\sigma : M_1 \rightarrow M$ ,  $\sigma(\gamma) = C \subset M$ , defined in a coordinate neighborhood  $U$  by the equations  $x^i = c^i(t)$ ,  $i = \overline{1, n}$  (the parameterization of  $C$  given by  $\Psi \circ \sigma \circ \Psi_\gamma^{-1}$ , where  $(U, \varphi) \in A$ ,  $(U_\gamma, \Psi_\gamma) \in A_\gamma$ ) and  $\gamma \subset U$ ,  $p_1, p_2 \in U$ . Let  $\varphi_{p_2 p_1} : T_{p_1} \rightarrow T_{p_2}$  be an isomorphism with the properties a), b), c) and  $c(t_0) = p_0, c(t) = p$ . Let  $V_0 \neq 0$  be a tangent vector  $V_0 \in T_{p_0}$  and  $\varphi_{p_0 p} \stackrel{\text{def}}{=} \varphi(t)$ ,  $V(t) = \varphi(t)V_0$ . We obtain the equation

$$\frac{dV}{dt} + NV = 0 \quad (4)$$

where  $N$  is the matrix  $\left( \frac{d\varphi}{dt} \varphi^{-1} \right)$ . From a), b), c) it follows that  $N$  depends on  $(C)$  and  $t$ . Since  $c \neq 0$  it is possible to consider the restriction  $N = N(x, \dot{x})$ . Hence we have, along  $C$

$$\frac{dV}{dt} + N(x, y)V = 0, \quad y = \dot{x} \quad (5)$$

This equation is an invariant over of the admissible changes of parameter  $t \rightarrow t'$  if the local coefficients  $\{N_k^i(x, y)\}$  of the matrix  $N$  are 1-homogeneous in  $y = \dot{x}$  (along of the curve  $C$ ). From the theory of the linear differential equations we have: the given vector  $V_0$  at  $p_0$  gives rise to a unique vector field  $V$  along  $C$ , with  $V(t_0) = V_0$ . Since  $y = \dot{x} \neq 0$  along  $(C)$ , the function 1-homogeneous  $(N)$  are not linear function in  $y$ , along  $(C)$ . We have  $N_k^i(x, y) \neq y^k \Gamma_{rk}^i(x)$  along  $C$ . For a change of coordinates  $t \rightarrow t'$ ,  $\{x^i\} \rightarrow \{x^{i'}\}$  we have (in  $p \in C, p \in U \cap U'$ )

$$N_{k'}^{i'}(x', y') \frac{\partial x^{k'}}{\partial x^k} = N_k^i(x, y) \frac{\partial x^{i'}}{\partial x^i} - \frac{\partial^2 x^{i'}}{\partial x^k \partial x^r} y^r \quad (6)$$

We have (6) for any two points  $p_1, p_2 \in M$  and for a piecewise smooth curve on  $M$ , joining  $p_1$  to  $p_2$ . If  $Osc^{(1)}M$  is identified to the total space  $E = TM$  and  $Osc^{(1)}-0 = TM-0 \stackrel{\text{def}}{=} \tilde{E}$  then using the above isomorphism a homogeneous non-linear connection  $N$  will be introduced. Using the equivalence relation  $(u, \pi^{-1}(U), N) \sim (\bar{u}, \pi^{-1}(U'), N')$  given by  $u = \bar{u}, \pi(u) = p \in M$  and (6) at each  $u$ , we get

**Definition 4.** An  $N$ -parallel transport along of the curve  $C$  on  $Osc^1M$  is defined by an isomorphism  $\varphi$  with the properties a), b), c), where the isomorphism  $\varphi$  is independent of the particular parametric representation of  $C$ .

Consequently we have a geometric interpretation of a 1-homogeneous non-linear connection as well as a natural prolongation of the geometry of the base  $M$  to the space  $Osc^{(1)}$ . For a more detailed treatment the reader is referred to R. Miron ([1], [5]). For the geometry of the space  $Osc^{(k)}$  we refer to the book of R. Miron ([4], [5]).

### 3 Some generalization

We consider the structure  $Osc^{(1)}M = \{Osc^{(1)}, \pi, M; N\}$  where  $N$  is a fixed non-linear connection. Hence  $H_uE, V_uE$  ( $u \in Osc^{(1)}E$ ) are isomorphic since they are both  $n$ -dimensional on obtain a general isomorphism  $\varphi : V \rightarrow H$  ( $\varphi_u : V_u \rightarrow H_u$  is a general isomorphism for every  $u \in E$ ). For  $Osc^{(1)}M$  we consider a  $\tau$ -system and an  $\Omega(\rho)$ -system. In [1] is defined the notion of the normal connection  $D$  on  $E = TM$  using  $\varphi$ . More general we set :

**Definition 5.** A linear connection  $D$  on  $E = TM$  satisfying the conditions

$$\varphi(vD_{hX}vY) = hD_{hX}\varphi(vY), \quad \varphi(vD_{vX}vY) = hD_{vX}\varphi(vY), \quad (7)$$

is called the almost normal connection  $\forall X, Y \in \mathcal{X}(M)$ .

If  $D$  is a  $d$ -linear connection then the almost normal connection  $D$  is called  $\varphi$ -normal connection. In particular if we consider the locally adapted basis  $(\delta/\delta x^i, \partial/\partial y^i)$  and the canonical isomorphism  $\varphi_0 : \partial/\partial y^i \rightarrow \delta/\delta x^i$  on obtain the definition for a normal connection by means of the local coefficients of a  $d$ -linear connection in the adapted basis.

**Definition 6.** A linear connection  $D$  on  $E$  satisfying

$$vD_{vX}\varphi(vY) = vD_{vX}\varphi(vX), \quad hD_{\varphi(vX)}vY = hD_{\varphi(vX)}vX, \quad (8)$$

is called a  $(q-d)$ -linear connection (or a quarter- $d$ -linear connection).

**Example.** A  $d$ -linear connection is a particular case of the  $(q-d)$ -linear connection. A  $\varphi$ -normal connection is a particular case of  $(q-d)$ -linear connection.

To study de classes of the almost connection and the classes  $\mathcal{D}(q-d; \varphi)$  of the  $(q-d)$ -linear connections, it is necessary to study the  $\tau$ -system and  $\Omega(\rho)$ -system. Using the algebra  $DT(TM)$  of a  $d$ -tensor fields on  $E = TM$  the general solutions of a  $\tau$ -system and of a  $\Omega(\rho)$ -system ([11]) are obtained. Using these solutions we obtain the theorem:

**Theorem 3.** The general solution of a  $\tau$ -system satisfying

$$\begin{aligned} h\tau(hX, \varphi(vY)) &= \varphi(v\tau(hX, vY)); \\ h\tau(vX, \varphi(vY)) &= \varphi(v\tau(vX, vY)), \quad \forall X, Y \in \mathcal{X}(TM) \end{aligned} \quad (9)$$

gives the classes  $\{D\}$  of the almost normal connections by  $\tau : D \rightarrow \overline{D}$ ,  $\overline{D}_X = D_X + \tau_X$ ,  $\forall X \in \mathcal{X}(M)$ ,  $\tau_X$  is a tensor field of type  $\binom{1}{1}$  and

$$\begin{aligned} \tau(X, Y) &= hA_1(hX, hY) + \varphi^{-1}(hA_1(hX, \varphi(vY))) + \\ &\quad + hA_2(vX, hY) + \varphi^{-1}(hA_2(vX, \varphi(vY))) \end{aligned} \quad (10)$$

where  $A_1, A_2$  are arbitrary tensor fields of type  $\binom{1}{2}$  on  $E = TM$ .

Evidently  $U_1(X, Y) = hA_1(hX, hY)$  is a  $d$ -tensor field of type  $\begin{pmatrix} 1 & 0 \\ 2 & 0 \end{pmatrix}$  and  $U_2(X, Y) = hA_2(vX, hY)$  is a  $d$ -tensor field of type  $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ .

**Theorem 4.** *The general solutions of a  $\Omega(\rho)$ -system satisfying*

$$h\Omega(hX, \varphi(vY)) - \varphi(v\Omega(hX, vY)) = \rho(\varphi(vY))hX \quad (11)$$

$$h\Omega(vX, \varphi(vY)) - \varphi(v\Omega(vX, vY)) = -\rho(vY)\varphi(vX) \quad (12)$$

is given by

$$\begin{aligned} \Omega(X, Y) = & \rho(vY)hX + \rho(hY)vX - \rho(\varphi(vY))\varphi^{-1}(hX) - \\ & -\rho(\varphi^{-1}(hY))\varphi(vX) + hA_1(hX, hY) + \varphi^{-1}(hA_1(hX, \varphi(vY))) + \\ & + \varphi(vA_2(vX, \varphi^{-1}(hY))) + vA_2(vX, vY). \end{aligned} \quad (13)$$

Using (10) and (13) we obtain:

**Theorem 5.** *The general solutions of a  $\tau$ -system satisfying*

$$\tau(X, Y) = \rho(X)Y + \rho(Y)X - \Omega(X, Y) \quad (14)$$

with  $\Omega$  satisfying (11), (12) is given by (14) where  $\Omega$  is given by (13).

Using these theorems are studied the special classes of the almost normal connections and the special classes of the  $\varphi$ -normal connections. In ([12]) using the above definitions are introduced the notion of Finsler hyperconnection on  $Osc^{(1)}M$ , which is an almost normal connection. Using the above theorems it is possible to study the connection transformations which preserve the classes of Finsler hyperconnections.

## 4 About the cotangent bundle

The geometry of total space of cotangent bundle  $\pi^* : T^*M \rightarrow M$  endowed with a non-linear connection  $N$  and the theory of Hamilton space were introduced by R. Miron using a special method. Because the space  $T^*M$  and its tangent bundle  $TT^*M$  have specific geometric structure which are totally different from the geometrical structure of  $TM$  and its tangent bundle  $TTM$  it follows that the geometry of  $T^*M$  is not a simple duality of the geometry of  $TM$ . Therefore the integrability of the structures  $(T^*M, (G, F^*))$  is more interesting and more complicated. This problem was solved by author in the paper [15]. The  $N$ -almost Hermitian model of the generalized Hamilton space of R. Miron is used. The applications to the homogeneous lift to cotangent bundle is obtained ([16]). Let  $H^n = (M, g^{rs}(x, p))$  be a generalized Hamilton space and  $H^{2n} = (T^*M, (G, F^*))$  the  $N$ -almost Hermitian model. If  $\Omega$  is the curvature of fixed non-linear connection on  $T^*M$  and  $\overset{*}{\Omega}$  is

the self-curvature of  $N$ ,  $(\overset{\star}{\Omega}(X, Y) = v\mathcal{N}_{F^*}(hX, hY), \forall X, Y \in X(T^*M)$  where  $\mathcal{N}_{F^*}$  is the Nijenhuis tensor of the almost complex structure  $F^*$ ) is obtained the main theorem.

**Theorem 6.**  $\omega = \overset{\star}{\Omega} - \Omega$  is an intrinsic invariants (i.e  $\omega$  depends only on the metric  $g$ ).

Because of this theorem  $\omega$  is called a non-holonomy distortion of the model  $H^{2n}$  relative to  $H^n$ . Using  $\omega$  we obtain:

**Theorem 7.** If  $\omega = 0$  then there exists a nondegenerate Hamiltonian  $H$ . The space  $(M, H)$  is an Hamilton space.

The modern geometry of higher order Lagrange spaces and of higher order Hamilton spaces was introduced by R. Miron ([4], [3]). Using this geometry it is possible to generalize the above theorem.

Another problem: Via Legendre transformation if  $L^{(k)}$  is an degenerate Lagrangian there exist a non-degenerate Lagrangian  $L^{(s)}$  such that  $H(L^{(k)}) = H(L^{(s)})$ , where  $H$  is the Hamiltonian associated to  $L^{(s)}$ ? Using the above theorem of R. Miron, if is solved this general problem then it is obtained a new method for the treatment of the degenerate Lagrangian of higher order. But this is an open problem.

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