MODELING THE INFORMATIONAL IMPACT ON THE DECISION-MAKING PROCESSES

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Abstract: To improve the accuracy, adequacy and effective functioning of economic systems at the macro and micro level it is necessary to develop reasonably efficient mathematical models for decision-making processes, which fully conform to the real situations. At present, information aspects in decision-making are not sufficiently studied. To improve the adequacy of the model it is necessary to implement analysis of the impact of information flows on the decision-making processes. In these article we develop mathematical models for decision-making processes in situations of risk, conflicts and various degree of decision-makers' awareness, using the mathematical theory of information extended games. We study the non informational extended games which are generated by the two directional flow informational extended strategies of the players. The theorem about the existence of the Nash equilibrium profiles in this type of the games is proved. The static game with informational extended strategies can be treated as a dynamic game with incomplete information and non informational extended strategies, where nature makes the first move, but not everyone observes nature's move.

1. Informational impacts in the game theory.

If we are going to introduce uncertainty in a game, we need to think clearly about what is known and what is not, and by whom. Some thing is common knowledge if every one knows it, every one knows that everyone knows it, everyone knows that everyone knows that everyone knows it etc. In standard game theory we make the following assumptions. The game structure (the tree) is common knowledge. Everyone perceives the same tree, and so knows what the possible moves are. In these article we analyze the impact of the access to information on a decision process. The most important problems in the field of information are the availability and the credibility. The reason is that some of the entities can achieve comparative advantage towards the others according to the additional information. Access to the additional, private information enables them not only to improve their decisions, but also to manipulate information. The problems with access to information generate conditions of perfect, imperfect, complete, incomplete, uncertain or asymmetric information. The sequence of moves can represent time but does not always have to. The information sets allow us to represent simultaneity in extensive form games. We assume that players never forget their own moves, or the fact that they have moved, or any thing that they once knew. So information sets can not link nodes with predecessors or successors. This property is known as perfect recall.

Perfect or imperfect information is a term used to describe a state of complete or incomplete knowledge about the actions of other players. Complete or incomplete information is a term used to describe a state of complete or incomplete knowledge about the normal form of the game, t. e. payoff functions, sets of strategies and others. In other words game of perfect information is one in which a player making a decision can always observe all previous decisions, so every information set contains a single decision node, as in contracting with observable proposal. In this case, everyone knows where they are in the game at all times. A game has imperfect information if there are information sets with more than one node so a player may be uncertain about where they are.

The complete information game is a game where each player knows exactly the rules. In other words each player starting the game knows the payoffs and strategies available to other players, while not necessarily has knowledge about the action of the other players inside the game. The games of incomplete information, known as I-games, were the fields of research for J. Harsanyi, Incomplete or asymmetric information in the game theory concerns the lack of access to information about the rival's strategies, payoffs or the payoffs' functions. Uncertain information is used to describe a game in which players do not know exactly what kind of game they are playing. They know what payoff of playing a particular strategy will be, given the strategies played by others.

There are two kinds of games with complete but imperfect information. The first are games with simultaneous moves, and the second are games where, late in the game, nature makes moves and those moves are not immediately revealed to all players. Moreover, any game of incomplete information can be transformed into a game of imperfect information by including nature as a player in the game and conditional payoffs for nature's unknown moves. According to the new definition of complete games from Rasmusen, in such games, nature is not allowed to make the first movement, although other players can observe this movement [1]. Simultaneous games and the games with the move of nature at some stage of the game, which is not observed immediately by players, are the games of complete and imperfect information. The concepts of incomplete information and of asymmetric information are not equivalent. However many games of incomplete information games are also games with the informational advantage of one player i.e. with asymmetric information. If there is no initial move by the nature, but the first player takes a move unobserved the second player, and the first player moves again later in the game, it is the game of asymmetric but complete information. It is connected with the principal-agent problem, when the agent possesses additional or private information, which is unavailable to the principal, not even at the end of the game. The game of incomplete (or asymmetric information) can be transformed into the game of uncertain information by applying the assumption, that nature chooses the sequence of player's movements. On the other hand the game of uncertain information can be transformed into the game of incomplete (or asymmetric information) by randomizing. Randomizing is ascribing expected probabilities to the different types of the rival or to his choices (strategies). The player has to determine probabilities according to available information, his previous experience, presumptions or intuition. As a result he eliminates nature from a game, so the game becomes a certain one. Game theory has developed in parallel with the theory of asymmetric information. As a result it influences asymmetric information research. Describing the problem of the access to the information in words of the game theory, symmetric information is a situation, in which none of the players has an informational advantage over the others. According to E. Rasmusen, in the game of symmetric information the informational set of a particular player in any node, where the player makes decision or in the end node contains at least the same elements that informational sets of other players. The information set for a particular player establishes all the possible moves that could have taken place in the game so far. In words of game theory the information set for player is a set of nodes available to one player in the given stage of the game, but on different paths. The player's information set includes only nodes at which this player moves. One node cannot belong to two different information sets of the player. Information sets also show the effects of unobserved moves by nature. If a game has complete information, each single node constitutes an information set. In fact it is the point that a player actually reached at the given stage of the game. In a game of incomplete information the information set consists of several nodes. However player does not know the exact location the game has reached in the game tree. In such a game it is often nature that moves the first.

Complete does not imply perfect information, but if background uncertainty is modeled as "moves by nature," perfect implies complete information.

So in the game theory the following type of information structures are used [1-3].

Complete information structure. All players know what game is being played. Specifically, the number of players, the actions available to each player, and the payoff associated with each action vector are common knowledge among the players

Incomplete information structure. A game with incomplete information tries to model situations in which some players have private information before the game begins. The initial private information is called the type of the player. The players' uncertainties is represented as a probability distribution over a set of possible games.

Complete and perfect information structure. Game with perfect information means that each player, when making any decision, is perfectly informed of all the events that have previously occurred i.e., each player observed the previous moves made by the other players. A game of perfect information is one in which there is no information set.

Complete and imperfect information structure. In the games with imperfect information the player has only partial information about the actions taken previously by another player, i.e., some of the previous moves by other players are not observed when a player is called upon to move.

2. Extensive form game. Preliminaries

A static game has a single stage, at which players make simultaneous decisions, as in contracting with unobservable proposal. A dynamic game has some sequential decisions, as in contracting with observable proposal. A strategy is a complete contingent plan for playing the game, which specifies a feasible decision for each of a player's decision nodes in the game and possible information states when he reaches them. In static games we sometimes say "decision" or "action" instead of "strategy.") A strategy is like a detailed chess textbook, not like a move. A player's feasible strategies must be independent of others' strategies, and specifying a strategy for each player determines an outcome (or at least a probability distribution over possible outcomes) in the game.

The strategic form is a convenient device for defining strategic equilibrium: it enables us to thin of the players as making single, simultaneous choices. However, actually to describe "the rules of the game," it is more convenient to present the game in the form of a tree. The extensive form is a formal representation of the rules of the game. It consists of a rooted tree whose nodes represent decision points (an appropriate label identifies the relevant player), whose branches represent moves and whose endpoints represent outcomes. Each player's decision nodes are partitioned into information sets indicating the player's state of knowledge at the time he must make his move: the player can distinguish between points lying in different information sets but can not distinguish between points lying in the same information set.

The difference between Static vs. Dynamic game is not that static game is represented by normal (strategic) form while dynamic game is represented by extensive form (game tree). As a matter fact, both can be represented by normal form or extensive form. The main difference between them is what is known by the players when they make their decision.

Formally, a rooted tree is a pair $(M;\sigma)$ where M is the finite set of nodes, and $\sigma : M \to M \cup \varnothing$ associates to each node its predecessor, i.e. the function that gives the node that comes immediately before any given node in the game tree. A (unique) node m_0 with no predecessors (i.e. $\sigma(m_0) = \varnothing$, is the root of the tree. Terminal nodes are those which are not predecessors of any node. Alternatively, we can say that m_t is a terminal node if there is no node $m \in M$ with $\sigma(m) = m_t$. Denote by $T(M)$ the set of terminal nodes.

Now we can give a formal definition of an extensive form game.

Definition. *An extensive form game consists of:*

1) $I=\{0,1,2,...,n\}$ a finite set of players;

2) A set of nodes and a rooted tree $(M;\sigma)$, where $M=\bigcup_{i\in I} M_i$;

3) A set o actions, denoted by A, and a labeling function $\alpha: M\setminus\{m_0\}\to \mathrm{A}$ where $\alpha(m)$ is the action at the predecessor of m that leads to m. If the value of the immediate predecessor function $\sigma(m)=\sigma(m')$ $\sigma(m)=\sigma(m')$ and $m\neq m$ then $\alpha(m)\neq\alpha(m')$. Here $m_0\in M$ is the initial node such that $\sigma(m_0)=\varnothing$.

4) A collection Π of information sets and $h: M\setminus T(M)\to\Pi$ a function that assigns for every node, except the terminal ones, which information set the node is in. So, the function h partitioned each M_i, $i\neq 0$, into information sets $P^i_1,...,P^i_{k_i}$, $M_i=\bigcup_{j=1}^{k_i} P^i_j$, $P^i_{j_1}\cap P^i_{j_2}=\varnothing$ for $j_1\neq j_2$ with the following condition: any two nodes x ,y from the same information set must have the same number of successors, and the set of move labels on the alternatives at x should coincide with the set of move labels on the at y. The interpretation is that when a player i has to choose an alternative at the node $m\in M_i$, he knows in what information set he is, but he does not know at what exact node from this information set he is making his choice.

5) *A function $\kappa:\Pi\to I\cup\{\varnothing\}$, where $\kappa(\Pi)$ is the player who moves at information set Π and then $\Pi_k=\{P\in\Pi\mid \kappa(P)=k\}$ be the information sets controlled by player $k\in I$..*

6) $\rho:\Pi_0\times\mathrm{A}\to[0,1]$ giving the probability that action $a\in\mathrm{A}$ is taken at the information set Π_0 of Nature.

g) $(u_1,...,u_i,...,u_n)$ with $u_i: T\to R$ being the payoff to player i.

This completes the definition of an extensive form game. Just a few more comments are in order at this stage. Let $C(m)=\{a\in\mathrm{A}\mid a=\alpha(m')\ \textit{for some } m' \textit{ with } \sigma(m')=m\}$ is the set of choices that are available at node *m*. Note that if *m* is a terminal node then $\mathrm{C(m)}=\varnothing$. If two nodes *m* and *m'* are in the same information set, that is, if $h(m)=h(m')$, then the same choices must be available at *m* and *m'*, that is $C(m)=C(m')$. Finally, we'll assume that Nature's information sets always have only one node in them.

3. Informational extended game and game with incomplete and imperfect information.

Unlike a strategic game, a Bayesian game is one in which some player does not know some parameter of the game they are playing. According to Harsanyi [4] any game of incomplete information can be transformed into a game of imperfect information (uncertainty about history of play). We explore models of incomplete information. Informally, a game of incomplete information is a game where the players do not have common knowledge of the game being played. This idea is tremendously important in capturing many economic situations, where a variety of features of the environment may not be commonly known. Among the aspects of the game that the players might not have common knowledge of are: payoffs; who the other players are; what moves are possible; how outcome depends on the action; what opponent knows, and what the knows I know etc.

A Bayesian game is a strategic form game with incomplete information. It consists of: a set of players $I=\{1,..,i,..,n\}$; an action set $\mathrm{A}_i, i\in I, \mathrm{A}=\prod_{i\in I}\mathrm{A}_i$; a type set $\Theta_i, i\in I, \Theta=\prod_{i\in I}\Theta_i$; a probability function $\rho_i:\Theta_i\times\Delta(\Theta_{-i})$ where $\Delta(\Theta_{-i})$ denote the set of all probability distributions on a set Θ_{-i}; a payoff function $u_i: A\times\Theta\to R$. The function ρ_i is summarizes what player *i* believes about the types of the other players given her type. So, $\rho_i(\theta_{-i}\mid\theta_i)$ is the conditional probability assigned to the type profile $\theta_{-i}\in\Theta_{-i}$ when the type of player *i* is θ_i. Similarly, $u_i(a|\theta)$ is the payoff of player *i* when the action profile is *a* and the type profile is θ. Here $a=(a_1,...,a_i,...,a_n)$ and $\theta=(\theta_1,...,\theta_i,...,\theta_n)$. A pure strategy for player *i* is a function which maps player *i*'s type into her action set $\mathrm{s}_i:\Theta_i\to\mathrm{A}_i$. $\mathrm{S}_i(\Theta_i)=\{s_i:\Theta_i\to\mathrm{A}_i\mid\forall\theta_i\in\Theta_i\ s_i(\theta_i)\in\mathrm{A}_i\}$ is the set of Bayesian pure strategies of the player *i*. To solve the game, we think of the players planning their decision rules at a stage before they observe their types. A Bayesian Nash equilibrium is a N.E. in the game of choosing decision rules. We use the following definition.

Definition. *A Bayesian strategy profile $\left(s^*_1,...,s^*_i,...,s^*_n\right)$ is a Bayesian Nash equilibrium if for all $i\in I$,*

$$s^*_i=\arg\max_{s_i\in S_i(\theta_i)}\sum_{\theta\in\Theta}u_i\left(s_i(\theta_i),s^*_{-i}(\theta_{-i}),\theta_i,\theta_{-i}\right)$$

or alternatively, for all $i\in I$, and realized type $\theta_i\in\Theta_i, a_i\in\mathrm{A}_i$;

$$\sum_{\theta_{-i}\in\Theta_{-i}}u_i\left(s^*_i(\theta_i),s^*_{-i}(\theta_{-i}),\theta_i,\theta_{-i}\right)p_i(\theta_{-i}\mid\theta_i)\geq$$

$$\geq\sum_{\theta_{-i}\in\Theta_{-i}}u_i\left(a_i,s^*_{-i}(\theta_{-i}),\theta_i,\theta_{-i}\right)\rho_i(\theta_{-i}\mid\theta_i),$$

where $\theta_{-i}=(\theta_1,...,\theta_{i-1},\theta_{i+1},...,\theta_n)$ and

$$s_{-i}(\theta_{-i})=(s_1(\theta_1),...,s_{i-1}(\theta_{i-1}),s_{i+1}(\theta_{i+1})...,s_n(\theta_n)).$$

So, Bayesian Nash Equilibrium is simply a Nash Equilibrium of the game where Nature moves first, chooses $\theta\in\Theta$ from a distribution with probability $\rho(\theta)$ and reveals θ_i to player *i*.

Let $\Gamma=\left\langle I;X_p;H_p:X\to R,p\in I\right\rangle$ be the strategic form of the static no cooperative games with complete and imperfect information. So the players know exactly your payoff functions and of the other players and they know the sets of the strategies. Players do not know what kind of the strategy will be chosen by the players. Here $I=\{1,2,...,n\}$ is the set of the players, X_p is a set of available alternative, H_p is the payoff function of the player $p \in I$ and $X = \prod_{p\in I} X_p$ is the set of strategy profiles for the game. In [5] it was studied the informational extensions of the games Γ, generated by a one-way directional informational flow, denoted by $j \to i$, which means: the player i, and only he, knows exactly what value of the strategy will be chosen by the player j and double-sided informational flow, denoted by $i \leftrightarrow j$ which means: at any time simultaneously player i knows exactly what value of the strategy will be chosen by the player j and player j knows exactly what value of the strategy will be chosen by the player i. We mention that the game is static, that is the players chose your informational extended strategies concomitantly, in other words, it is no significant the order of the chosen strategies. The players do not known the informational type of the other players, so the player i (respectively j) do not know that the player j (respectively i) knows what value of the strategies will be chosen. The set of the informational extended strategies of the player i (respectively j) is the set of the functions $\Theta_i = \{\theta_i : X_j \to X_i\}$ (respectively $\Theta_j = \{\theta_j : X_i \to X_j\}$) so that $\forall x_j \in X_j, \theta_i(x_j) \in X_i$ and $\forall x_i \in X_i, \theta_j(x_i) \in X_j$. So according to [5] we consider the following informational extended game $\Gamma(i \leftrightarrow j) = \left\langle I, \Theta_i, \Theta_j, \{X_p\}_{p\in I\setminus\{i,j\}} \{\widetilde{H}_p\}_p \right\rangle$ with uncertainty about history of play.

In this article we study the case when the informational strategies of the players already was chosen and so appear the necessity to study the informational not extended game generated by the chosen informational extended strategies. These games differ in: a) the sets of the strategies that are the subsets of the sets of strategies in the initial no extended informational game; b) how the payoff functions of the players will be constructed. The game with the following normal form $\Gamma(\theta_i,\theta_j) = \left\langle I, \{X_p\}_{p\in I} \{\widetilde{H}_p\}_p \right\rangle$ where $\widetilde{H}_p(x_i,x_j,x_{-ij}) \equiv H_p(\theta_i(x_j),\theta_j(x_i),x_{-ij})$, will be called informational non extended game generating by the informational extended strategies θ_i, θ_j of the $i \leftrightarrow j$ informational extended game. We can proof the following theorem.

Theorem. *Let the game $\Gamma(\theta_i,\theta_j)$ satisfying the following conditions: 1) the X_p is the non-empty, compact and convex subset of a some finite dimensional Euclidean space for all $p \in I$; 2) the functions θ_i, θ_j are continuous on X_i and X_j for all $p \in I$ and functions H_p are continuous on X_p for al $p \in I$ l; 3) the functions θ_i (corresponding θ_j) is quasi-concave on X_j (corresponding on X_j), functions H_p are quasi-concave on $p \in I\setminus\{i,j\}$ and monotonically increasing on $X_i \times X_j$. Then $NE\|(\Gamma(\theta_i,\theta_j))\| \neq \varnothing$.*

The game $\Gamma(i \leftrightarrow j)$, and in particular the game $\Gamma(\theta_i,\theta_j)$, will be treated as the Bayesian game because all informational extended strategies, for example $\theta_i \in \Theta_i$, of the player i generate the uncertainty of the player j about the complete structure of the payoff function $H_p(\theta_i(x_j),\theta_j(x_i),x_{-ij})$ in the game with non informational extended strategies. So player i and j does not know some parameter of the game they are playing, notably the payoff functions in case if they use the strategies $x_i \in X_i$ and $x_j \in X_j$.

For solving the game $\Gamma(i \leftrightarrow j)$ with informational extended strategies we can apply the main results of the Bayesian game theory and the fundamental observation of the Harsanyi: the games of incomplete information can be thought of as games of complete but imperfect information where nature makes the first move and selecting the informational extended strategies $\theta_i \in \Theta_i$ and $\theta_j \in \Theta_j$. Finally, using the Harsanyi's transformation for solving the informational extended game $\Gamma(i \leftrightarrow j)$ or $\Gamma(\theta_i,\theta_j)$ we can solve the Bayesian game with non informational extended strategies where $I = \{0,1,...,p,...n\}$, the action set $A_i \equiv X_i \ \forall i \in I\setminus\{0\}$, the type set of the player i and j will coincide with the choose informational extended strategies $\theta_i \in \Theta_i$ (corresponding $\theta_j \in \Theta_j$) and the payoff functions of the players are $\widetilde{H}_p(x_i,x_j,x_{-ij})$ for all $p \in I\setminus\{0\}$.

4. References

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